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Maximum Size of Social Security in a Model of Endogenous Fertility

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Abstract

Social security tends to be unsustainable in nature. It reduces individuals' demand for children as a measure to support their lifestyle during old age, which in turn undermines the financial basis of social security. Using a simple overlapping generations model with endogenous fertility and income transfer from children to parents, we discuss the maximum size of a pay-as-you-go social security program that can prevent a cumulative reduction of fertility and make a program sustainable. We also show that a child-care allowance raises the maximum size of the program and raises an individual's lifetime utility.

Key words: social security, fertility, intergenerational income transfer

JEL classification codes: H31, H55

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1. Introduction

Declining fertility puts strong pressures on the sustainability of social security. Most advanced countries have instituted pay-as-you-go (PAYG) social security programs, which rely heavily on contributions from young and future generations. Because a decreasing number of children is most likely to make programs less sustainable, many policymakers now call for child-care support, which is expected to prevent fertility from declining further.

However, social security tends to be unsustainable or even self-destructive in nature. The old-age security hypothesis, which treats children as capital goods for the material support during old age, implies that social security reduces demand for children (Zhang and Nishimura, 1993). This is also the case if we interpret a PAYG program as insurance against not having children (Sinn, 2004). Social security provides older individuals with financial support, at least partially substituting children. A reduced motive for having children reduces fertility and renders the financial base of social security vulnerable.

The old-age security hypothesis holds more in developing countries than in developed ones. Furthermore, many preceding analyses of endogenous fertility have interpreted children as consumption goods—that is, they have included the number of children in an individual's utility function—and/or incorporated altruistic motives, following seminal works of Becker and Barro (1988) and Barro and Becker (1989). Moreover, various recent studies have examined the effectiveness of child-care support to mitigate the negative impact of low fertility on social welfare and social security (Groezen, Leers, and Meijdam, 2003; Fenge and Meier, 2005; Hirazawa and Yakita, 2008).

The negative feedback loop between social security and fertility, which is inherent in social security, must not be ignored, especially if sustainability of social security confronts an imminent risk under conditions of declining fertility. In this study, we explicitly address the risk of a cumulative reduction in fertility and discuss how to prevent social security from collapsing, exclusively examining the role of children as capital goods for support during old age. To this end, we explore a simple overlapping-generations model with endogenous fertility and income transfer from children to parents. Incorporating the old-age gift into the model of endogenous fertility, Zhang and Zhang (1998) and Wigger (1999) show that social security programs, if small sized, can stimulate per capita income growth, but not if they are too large. We extend their analysis to examine, explicitly, the maximum size of social security that can prevent fertility from cumulatively declining and prevent social security from collapsing.

We also show that introducing a child-care allowance expands the maximum size of social security, a reasonable result given an expected positive effect on fertility of child-care support. Moreover, we compare the impact on an individual's utility and fertility of two policies to show that social security reduces fertility and utility, whereas the child-care allowance raises them.

The remainder of this paper is constructed as follows. **Section 2** presents a basic model and discusses the benchmark state that exists before introducing social security. **Section 3** introduces a PAYG social security program, and **Section 4** adds a child-care allowance to it. Each of these two sections examines the dynamics of fertility and necessary conditions to make social security sustainable. **Section 5** presents graphical illustrations of the results in model analysis. **Section 6** concludes this presentation.

2. Before introducing social security

We consider a simple overlapping generations model, in which individuals live in two life periods, respectively, when they are young and old. Individuals treat children solely as capital goods for material support during their old age; there are no altruistic motives. We start with the case in which no social security program exists. Each individual maximizes lifetime utility.

$$u = u(c_1, c_2) = \gamma \ln c_1 + (1-\gamma)\ln c_2, \;\; 0 < \gamma < 1 \qquad (1)$$

Therein, c_1 and c_2 respectively signify consumption in young and old age periods. The budget constraints are given as

$$c_1 = [1-\theta - c(n)]w - s,$$
$$c_2 = (1+r_{+1})s + \theta w_{+1} n,$$

for each life stage, where s, w, r, n, θ, and $c(n)$ represent savings, wages, the interest rate, the number of children, gifts to parents, and the cost function of childrearing. The suffix "+1" indicates one period ahead. In addition, both θ and $c(n)$ are defined in terms of the ratio to wage. When young, an individual earns wage income, bears some children, and gives some pecuniary or material gifts to their old parents. When old, individuals rely on their own savings and gifts from their children. No bequest[1] exists. We also assume for simplicity that an individual perfectly foresees w_{+1} and r_{+1}.

As for the old-age gift ratio θ, individuals choose its optimal value to maximize their lifetime utility, assuming that their children will make the same choice as their own if other variables remain unchanged. We assume that old parents take the value of the gift received from their children as given, even if it differs from what they expected to receive from their children. In the equilibrium, each generation calculates the optimal gift such that each generation gives the same fraction of their own wage income and no generation has an incentive to change the size of the gift (see Zhang and Zhang, 1998).

The cost function of childrearing is specified as

$$c(n) = cn^{\varepsilon}, \quad c > 0, \tag{2}$$

where ε is the elasticity of the cost of childrearing with respect to the number of children. Moreover, we assume $\varepsilon > 1$.

The first-order conditions for utility maximization are given as

$$u_1 = (1+r_{+1})u_2,$$
$$c'(n)wu_1 = \theta w_{+1} u_2,$$
$$wu_1 = w_{+1} n u_2$$

with respect to saving, the number of children, and the gift ratio, respectively, where $u_i = \partial u / \partial c_i$. From these three conditions, we have

[1] As noted later, we can discuss (non-altruistic) income transfer from parents to children by replacing θ with $-\theta$, while keeping the main results unchanged.

$$\frac{\theta w_{+1}}{c'(n)w} - 1 = \frac{w_{+1}n}{w} - 1 = r_{+1}, \tag{3}$$

which means that the rates of return from childrearing, the old-age gift, and saving are all equalized in utility maximization. This condition (3) is reduced to

$$\frac{\theta}{c'(n^*)} - 1 = n^* - 1 = r^* \tag{4}$$

in the steady state, where n^* and r^* are the steady-state number of children and interest rate.[2]

If (3) holds, then (i) the lifetime budget constraint is reduced to

$$c_1 + \frac{c_2}{1 + r_{+1}} = [1 - c(n)]w, \tag{5}$$

(ii) the old-age gift ratio is given as

$$\theta = \varepsilon c(n), \tag{6}$$

using (3), and (iii) the optimal saving is calculated as

$$s = [1 - \theta - c(n)]w - \gamma[1 - c(n)]w = [1 - \gamma - (1 - \gamma + \varepsilon)c(n)]w. \tag{7}$$

The wage income and the interest rate are derived from the competitive firms' profit maximization. Assuming that the production function is given as

$$y = k^\alpha, \quad 0 < \alpha < 1,$$

where k is the capital–labor ratio and that capital stock fully depreciates in one life period, then we have

$$w = (1 - \alpha)k^\alpha, \quad 1 + r = \alpha k^{\alpha - 1}. \tag{8}$$

The market equilibrium for capital (and for goods) is expressed as

$$k_{+1} = \frac{s}{n}. \tag{9}$$

Then, combining (3), (7), (8), and (9) yields the fertility equation:

$$c(n) = \frac{(1 - \alpha)(1 - \gamma) - \alpha}{(1 - \alpha)(1 - \gamma + \varepsilon)}. \tag{10}$$

Normalizing the number of children before introducing social security as unity, we

[2] This resultant fertility is determined solely by the interest rate, giving basically the same result as that of Becker and Barro (1988), who incorporate altruistic bequests in the model of endogenous fertility.

have the equation shown below.

$$c = \frac{(1-\alpha)(1-\gamma)-\alpha}{(1-\alpha)(1-\gamma+\varepsilon)} \tag{11}$$

3. Introducing social security

This section introduces a PAYG social security program, by which a young individual pays the social security tax of $t\times100$ percent of wages and an older individual receives the benefit with a replacement ratio of $\beta\times100$ percent of the wage paid to the young individual. Therefore, the lifetime budget constraints are given as

$$c_1 = [1-\theta-c(n)-t]w - s,$$
$$c_2 = (1+r_{+1})s + (\theta n+\beta)w_{+1}.$$

Because the number of children, the old-age gift ratio, and savings are adjusted in the same way as before introducing social security, condition (3) holds here again. Consequently, the lifetime budget constraint is expressed as

$$c_1 + \frac{c_2}{1+r_{+1}} = \left[1-c(n)-t+\frac{\beta}{n}\right]w, \tag{12}$$

and the optimal saving is given as

$$s = \left[1-\gamma-(1-\gamma+\varepsilon)c(n)-(1-\gamma)t-\frac{\gamma\beta}{n}\right]w. \tag{13}$$

Meanwhile, the budget constraint for the government is given as

$$t = \frac{\beta}{n_{-1}}, \tag{14}$$

where the government first sets up the replacement ratio β; then it adjusts the tax rate t to balance the PAYG social security program at each period, taking the observed number of the current young individuals n_{-1} as given.

Then, combining (3), (8), (9), (13), and (14) yields the dynamic equation of fertility:

$$c(n) = \frac{(1-\alpha)(1-\gamma)-\alpha}{(1-\alpha)(1-\gamma+\varepsilon)} - \frac{\beta}{1-\gamma+\varepsilon}\left(\frac{\gamma}{n}+\frac{1-\gamma}{n_{-1}}\right).$$

Normalizing the number of children before introducing social security as unity and using (11), we have

$$n^{\varepsilon} = 1 - \left(\frac{\gamma}{n} + \frac{1-\gamma}{n_{-1}} \right) \frac{\beta}{A}, \tag{15}$$

where

$$A \equiv \frac{(1-\alpha)(1-\gamma)-\alpha}{1-\alpha} > 0 .$$

From (15), we confirm that the number of children continues to decline after introduction of social security. Introducing social security reduces the demand for children as capital goods used for material support during old age and correspondingly reduces the cost of child rearing, which also engenders a reduction in the old-age gift (see (6)). Consequently, individuals can increase saving, which accelerates capital accumulation and reduces the interest rate. This brings a reduction in the rate of return from childrearing (see (3)) and engenders a further reduction in fertility. Under this adjustment, old parents depend less on the gifts from their children than before introducing social security because they receive social security benefits.

Next, we consider the maximum size of social security that can prevent a cumulative reduction in fertility and make social security sustainable. From (15), the equation which solves the steady-state number of children, n^*, is expressed as

$$n^{*\varepsilon} = 1 - \frac{\beta}{An^*}. \tag{16}$$

To consider the solutions of this equation graphically, **Figure 1** depicts curves of $f(n^*)=n^{*\varepsilon}$ and $g(n^*)=1-\beta/(An^*)$. This figure suggests that an overly high value of β engenders no steady-state solution of n^* because, thereby, the $g\,(n^*)$ curve is shifted downward and located below the $f\,(n^*)$ curve. The maximum value of β, denoted by β_+, is such that makes the two curves mutually tangent. Considering $f(n^*)=g(n^*)$ and $f'(n^*)=g'(n^*)$, we calculate

$$\beta_+ = \varepsilon (1+\varepsilon)^{-(1+\varepsilon)/\varepsilon} A ,$$

which engenders $n=(1+\varepsilon)^{-1/\varepsilon}$. Simple calculations show that β_+ is an increasing function of ε and a decreasing function of α and γ. The number of children continues falling cumulatively and the social security program collapses if β exceeds β_+. Consequently, we can state the following.

Proposition 1. *A PAYG social security program should be maintained within a certain limited size to prevent a cumulative reduction of fertility and a collapse of the program.*

Another interesting question to be addressed is whether introducing social security raises an individual's lifetime utility. We concentrate on the steady state, in which the lifetime budget constraint (12) is reduced to (5), the same as before introducing social security. In addition, an individual's adjustment equalizes the rate of return from the old-age gift to that from saving; that is, $n^*-1=r^*$, which makes the net rate of return from PAYG social security equal to zero. Therefore, social security affects the lifetime budget and utility entirely through its impact on fertility. Because the level of utility in the steady state, u^*, is given as

$$\begin{aligned} u^* &= \gamma \ln\left[\left(1-c\left(n^*\right)\right)w^*\right] + \left(1-\gamma\right)\ln\left[n^*\left(1-c\left(n^*\right)\right)w^*\right] \\ &= \ln w^* + \ln\left[1-c\left(n^*\right)\right] + \left(1-\gamma\right)\ln n^*, \end{aligned}$$

we have

$$\frac{du^*}{d\beta} = \left[\frac{1}{w^*}\frac{dw^*}{dn^*} - \frac{c'\left(n^*\right)}{1-c\left(n^*\right)} + \frac{1-\gamma}{n^*}\right]\frac{dn^*}{d\beta}. \tag{17}$$

We can show $dn^*/d\beta<0$ as long as social security stays sustainable ($\beta \leq \beta_+$) from (16) and also [ ] >0 in the RHS of (17), as long as

$$n^* < \left[\frac{\left(1-\alpha\right)\left(1-\gamma+\varepsilon\right)}{\left(1-\alpha\right)\left(1-\gamma+\varepsilon\right)-\alpha}\right]^{1/\varepsilon}, \tag{18}$$

using $dw^*/dn^* = -(\alpha/n^*)^{1/(1-\alpha)}<0$, $c'(n^*)=c\varepsilon n^{*\varepsilon-1}>0$ and (11). We start with $n^*=1$. Therefore, (18) holds and so we have $du^*/d\beta<0$. Consequently, we can state the following.

Proposition 2. *A PAYG social security program reduces an individual's lifetime utility in the presence of income transfer from children to their parents.*

This proposition is consistent with a conventional view that a PAYG social security program reduces lifetime utility under declining fertility. It is noteworthy, however, that the negative impact of social security on utility is not caused by a reduction of lifetime income in our model. Indeed, social security *raises* lifetime income because it makes individuals reduce the number of children and increase saving, which in turn accelerates capital accumulation and raises per-capita income. At the same time, however, it reduces the interest rate, which directly implies a higher cost of old-age consumption. The negative sign of eq. (17) indicates that this negative effect dominates positive effects from lower fertility—that is, an increase in lifetime income and a reduction in the childrearing cost—and reduces net lifetime utility.[3]

4. Adding a child-care allowance

This section introduces a child-care allowance in addition to social security, a reasonable response to declining fertility. Presuming that the government subsidizes $\varphi\times100$ percent of child-rearing cost and finances it by levying a wage-proportional tax of $v\times100$ percent on young individuals, then the budget constraints in two life stages are given as

$$c_1 = \left[1-\theta-(1-\varphi)cn-t-v\right]w-s,$$
$$c_2 = (1+r_{+1})s+(\theta n+\beta)w_{+1}.$$

The number of children, the old-age gift ratio and savings are adjusted such that

$$\frac{(1-\varphi)\theta w_{+1}}{c'(n)w}-1=\frac{w_{+1}n}{w}-1=r_{+1}, \tag{19}$$

and the lifetime budget constraint is reduced to

$$c_1+\frac{c_2}{1+r_{+1}}=\left[1-(1-\varphi)c(n)-t-v+\frac{\beta}{n}\right]w; \tag{20}$$

the optimal saving is given as

$$s=\left[1-\gamma-(1-\varphi)(1-\gamma+\varepsilon)c(n)-(1-\gamma)t-(1-\gamma)v-\frac{\gamma\beta}{n}\right]w. \tag{21}$$

Meanwhile, the government faces two budget constraints: the first is

[3] Zhang and Zhang (1998) showed that higher social security taxes lead to higher growth. This is consistent with our result, which shows that social security reduces the old-age gift and increases saving. However, higher growth does not mean higher utility, as suggested by Proposition 2.

$$v = \varphi c(n) \tag{22}$$

for the child-care allowance and the second is (14) for social security.

Then, combining (8), (9), (14), (21), and (22) yields

$$c(n) = \frac{(1-\alpha)(1-\gamma)-\alpha}{(1-\alpha)[1-\gamma+(1-\varphi)\varepsilon]} - \frac{\beta}{[1-\gamma+(1-\varphi)\varepsilon]}\left(\frac{\gamma}{n}+\frac{1-\gamma}{n_{-1}}\right).$$

Normalizing the number of children before introducing social security as unity and using (10), we obtain

$$n^{\varepsilon} = B(\varphi)\left[1-\left(\frac{\gamma}{n}+\frac{1-\gamma}{n_{-1}}\right)\frac{\beta}{A}\right], \tag{23}$$

where

$$B(\varphi) = \frac{1-\gamma+\varepsilon}{1-\gamma+(1-\varphi)\varepsilon} \geq 1. \tag{24}$$

The equation, which gives the steady-state solution for the number of children, n^{**}, is given as

$$n^{**\varepsilon} = B(\varphi)\left[1-\frac{\beta}{An^{**}}\right]. \tag{25}$$

It might be readily apparent from (24) and (25) that a child-care allowance raises fertility. The maximum value of β, which is denoted by β_{++} is calculated as

$$\beta_{++}(\varphi) = [B(\varphi)]^{1/\varepsilon}\beta_{+} \geq \beta_{+}$$

using simple calculations like those used for β_{+}.

Finally, we consider the impact of child-care allowance on an individual's lifetime utility, particularly addressing the steady state. As before introducing child-care allowance, the lifetime budget constraint (20) is reduced to (5). Consequently, the impact on individual utility of social security in the steady state is calculated as

$$\frac{du^{**}}{d\varphi} = \left[\frac{1}{w}\frac{dw^{**}}{dn^{**}} - \frac{c'(n^{**})}{1-c(n^{**})} + \frac{1-\gamma}{n^{**}}\right]\frac{dn^{**}}{d\varphi},$$

in the same way as (17). Assuming that (18) (with replacing n^{*} with n^{**}) holds and considering $dn^{**}/d\varphi>0$ from (24) and (25), we have $dn^{**}/d\varphi>0$. Therefore, we can state the following.

Proposition 3. *The child-care allowance raises fertility, expands the maximum size of the PAYG social security program, and raises an individual's lifetime utility in the presence of income transfer from children to their parents.*

A child-care allowance will reduce per-capita income because of the larger number of children, but it raises the interest rate and reduces the cost of old-age consumption. The latter effect dominates the former and reduces the net lifetime utility, which is an opposite outcome from that of introducing social security.

5. Graphical illustration

This section graphically presents the results described in **Sections 3** and **4**. We tentatively assume γ=0.5, α=0.25, and ε=2. Normalizing the number of children before introducing social security as unity, we have c=0.0667 from (11) and A=0.0167. Then, the dynamics of fertility, (15), is expressed as

$$n = \sqrt{1 - 3\left(\frac{1}{n} + \frac{1}{n_{-1}}\right)\beta},$$

and the maximum size of social security is calculated as β_{+}=0.0642.

Figure 2 portrays the dynamics of fertility, with the $n=n(n_{-1})$ curve and the 45-degree line along which steady states must lie, for three difference replacement ratios, β=0.05, 0.0642(=β_{+}), and 0.07. If β=0.05, the $n=n(n_{-1})$ curve crosses the 45-degree line at n=0.787 and n=0.319, which correspond to stable and unstable state solutions, respectively. Assuming that the economy starts at n=1, the number of children will fall to and stabilize at 0.787. When β is raised to 0.0642, the curve becomes tangent with the 45-degree line and yields the only stable number of children of n=0.579. When β is 0.07, which is greater than 0.0642, the curve does not cross the 45-degree line, suggesting that the number of children falls cumulatively to zero.

We then add the child-care allowance to social security, considering two different values of child-care allowance: φ=0.25 and 0.5. The new maximum size of social security is calculated as β_{++}=0.0717 when φ=0.25, and β_{++}=0.0828 when φ=0.5, both of which are higher than β_+=0.0642, confirming that the child-care allowance raises the maximum size of social security.

Figure 3 depicts the dynamics of fertility, setting β=0.07 greater than 0.0642(=β_+). Without the child-care allowance (φ=0), the curve does cross the 45-degree line, meaning that social security is not sustainable, as described above. Setting φ=0.25, the curve crosses the 45-degree line and the number of children is stabilized at n=0.725. With φ=0.5, the number of children rises to 0.973.

6. Conclusion

We discussed how to make a pay-as-you-go social security sustainable, based on a simple overlapping-generations model with endogenous fertility and an old-age gift from children to parents. Social security is unsustainable in nature, in that it reduces individuals' demand for children as a measure to support their old-age life, which tends to undermine the financial basis of social security.

The key results from our analysis are summarized as follows. First, a PAYG social security program should not be too large, because of the risk that a large program leads to a cumulative reduction in fertility. To make the program sustainable, we should contain its size within a certain limit, as determined by parameters related to individual utility, production, and the cost of childrearing functions.

Second, a PAYG social security program reduces an individual's lifetime utility. In response to an introduction of social security, individuals reduce the number of children and the old-age gift to their parents, which raises saving and raises per-capita income. However, the lowered interest rate raises the cost of old-age consumption and reduces the net lifetime utility by more than offsetting the positive effects of lower fertility.

Third, the child-care allowance raises the maximum size of the social security program and helps enhance the sustainability of the program by encouraging individuals to rear children. Furthermore, an increasing number of children raises utility because it reduces the cost of old-age consumption by reducing savings and raising the interest rate.

These results hold even if we consider the opposite direction of intergenerational transfer, i.e., bequests from old parents to children, as far as an individual's utility does not include an altruistic aspect. Indeed, replacing θ with $-\theta$ leaves the story mostly intact. In this setup, individuals increase income transfer to their children to offset its impact on lifetime income in response to an introduction of social security. Consequently, social security affects an individual's utility entirely through fertility, just as in the case of the old-age gift.

Including another aspect of children, especially their role as consumption goods, and altruistic motives will most likely engender mixed results. However, the inherent unsustainability of old-age social security cannot be alleviated completely so long as social security programs at least partially substitute intergenerational income transfer.

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Figure 1. Solving the steady-state number of children

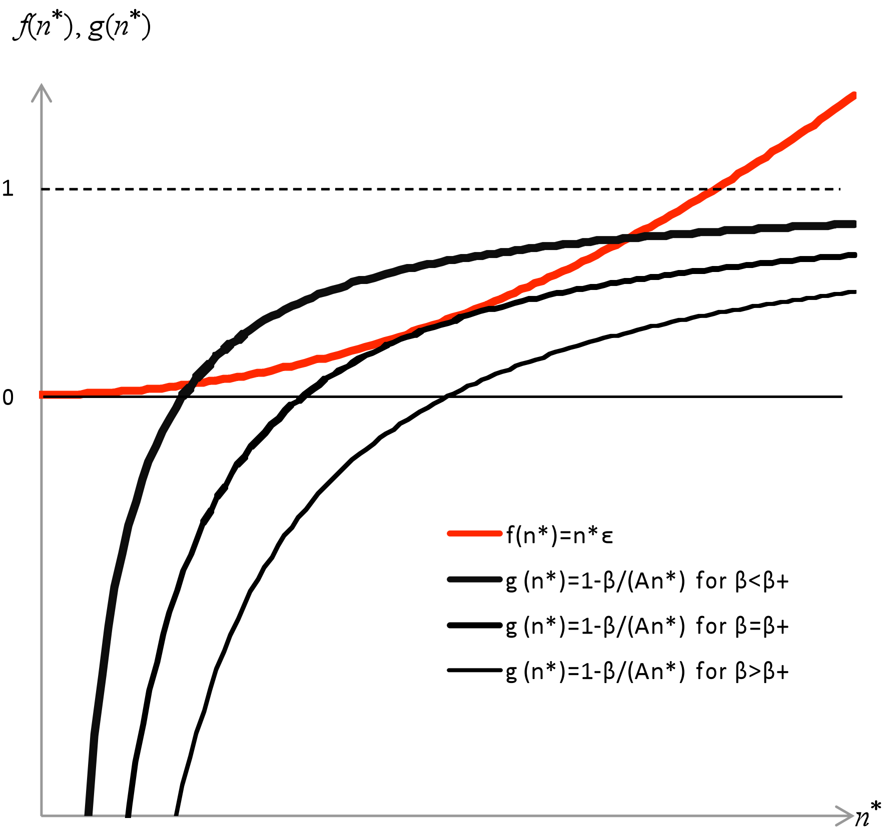
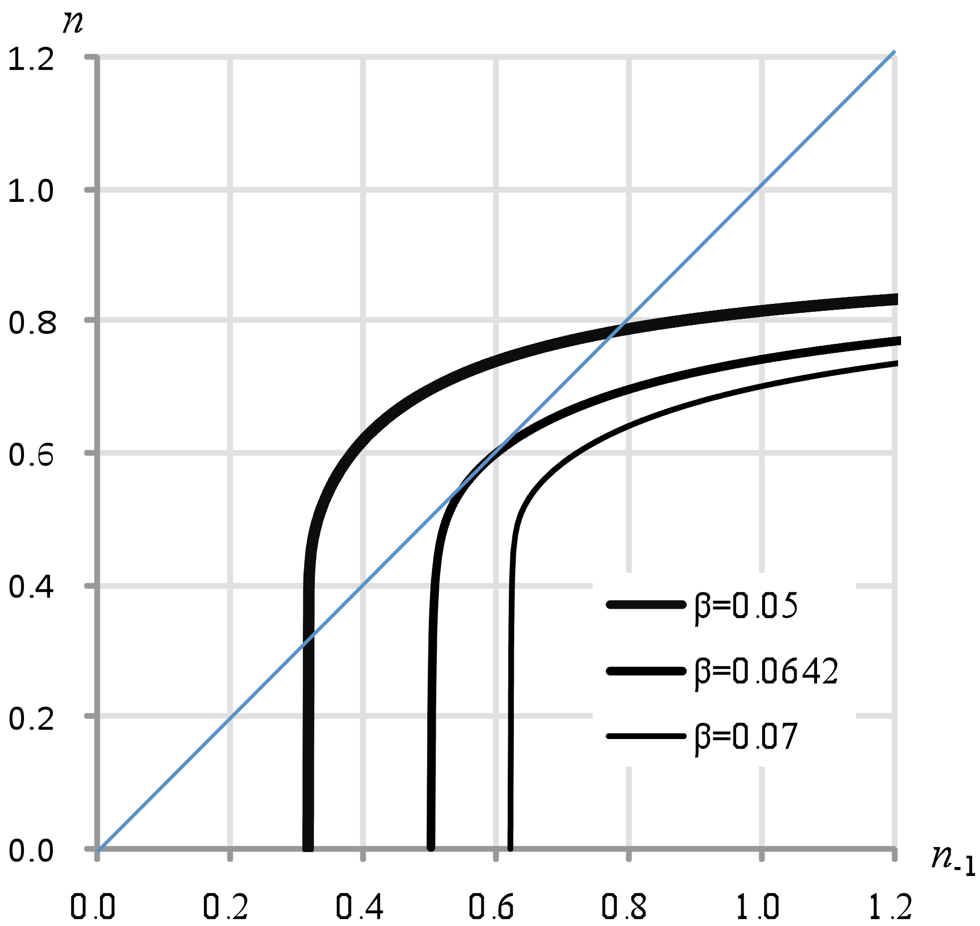
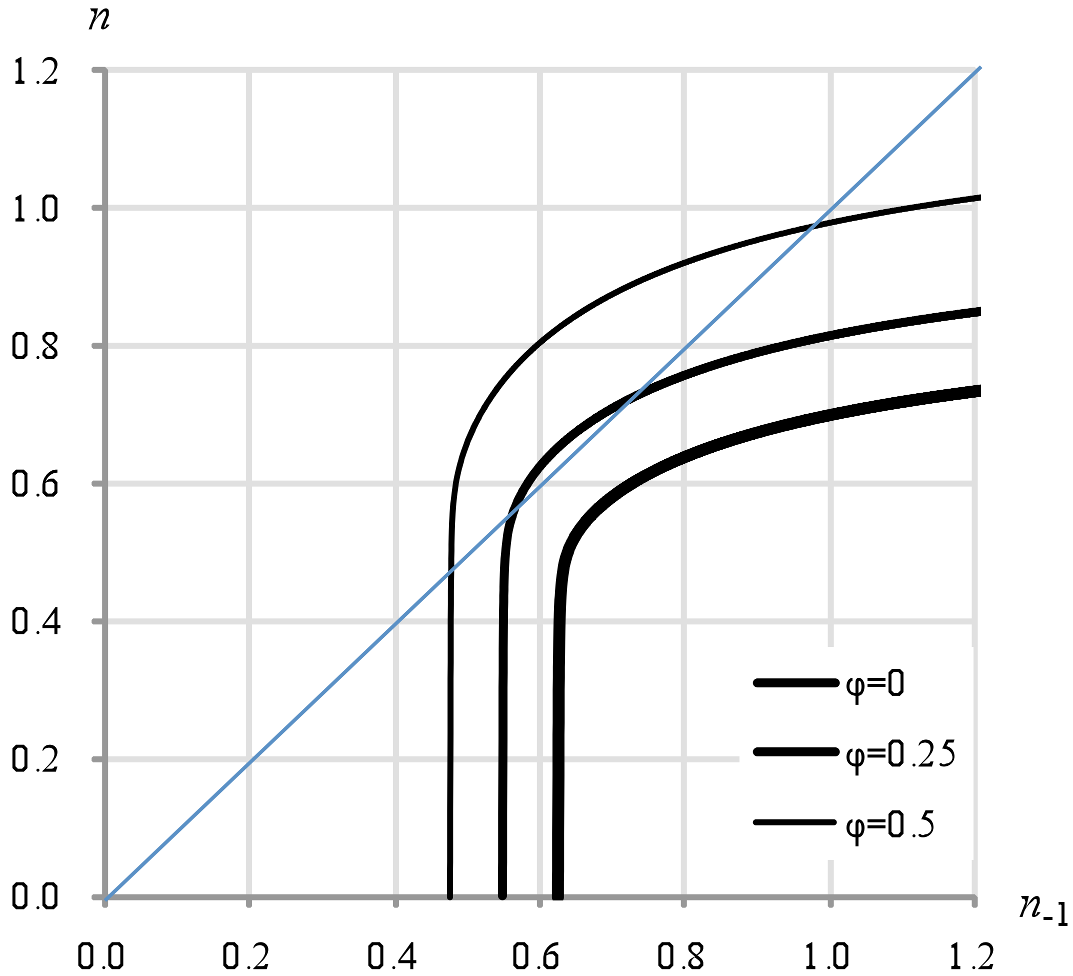


Figure 2. Dynamics of fertility under social security



Note: γ=0.5, α=0.25, and ε=2 are assumed.

Figure 3. Dynamics of fertility under social security and child-care allowance



Note: β=0.07, γ=0.5, α=0.25, and ε=2 are assumed.