



## DISCUSSION PAPERS ON DEMOGRAPHY AND POLICY

#2009-003

Child Benefit and Fiscal Burden:  
OLG Model with Endogenous Fertility

Kazumasa Oguro  
Institute for International Policy Studies

Junichiro Takahata  
Graduate School of Economics,  
Hitotsubashi University

Manabu Shimasawa  
Akita University

September 2009



HITOTSUBASHI UNIVERSITY

Public Economics Program  
School of International and Public Policy  
Tokyo, Japan

# Child Benefit and Fiscal Burden: OLG Model with Endogenous Fertility<sup>1</sup>

## *Abstract*

In this paper, we present an OLG simulation model with endogenous fertility in order to analyze the relationship between child benefit and fiscal burden in Japan.

Our simulation results show that expansion of the child benefit will improve the welfare of current and future generations. On the other hand, our findings show that we cannot expect a significant long-term improvement in welfare solely from implementing a policy of increasing the consumption tax.

If both the sustainability of the fiscal budget and the improvement of the welfare of current and future generations are requirements, we will need to promote a strategy consisting of such components as a policy-mix that includes both child benefit expansion and additional fiscal reform, i.e. increasing the consumption tax. Implementation of such a policy-mix could be expected to yield a higher economic level in the welfare of current and future generations than could be expected solely from consumption tax reform.

*Keywords:* Computable general equilibrium (CGE) model; overlapping generations (OLG); child benefit; endogenous fertility

*JEL Classification:* C68; D9; E62; H5; H6; J13

Kazumasa Oguro  
*Consulting Fellow, Research  
Institute of Economy, Trade and  
Industry,  
Ministry of Economy, Trade and  
Industry, Government of Japan  
and  
Research Fellow, Institute for  
International Policy Studies  
[ZVU07057@nifty.com](mailto:ZVU07057@nifty.com)*

Junichiro Takahata  
*Graduate School of Economics,  
Hitotsubashi University  
and  
Researcher, Policy Research  
Institute, Ministry of Finance,  
Government of Japan  
[ed042004@g.hit-u.ac.jp](mailto:ed042004@g.hit-u.ac.jp)*

Manabu Shimasawa  
*Associate Professor,  
Akita University, Japan  
[mshimasawa@ed.akita-u.ac.jp](mailto:mshimasawa@ed.akita-u.ac.jp)*

---

<sup>1</sup> This paper is a revised version of one presented at the Investigative Meeting of IPSS Discussion Paper on June 17, 2009. We are grateful for helpful comments and discussions from Toshiyuki UEMURA, Masato JINNO, and the meeting participants. Finally, the opinions expressed in this paper are those of the authors and do not represent the views of the organizations to which we belong. Any remaining errors are the responsibility of the authors.

## 1. Introduction

Developed countries currently face unprecedented demographic changes which require extensive reform in fiscal systems, social security systems, and other related programs. However, due to conflicting interests between younger and older generations, reform may be restricted. As an example of a pay-as-you-go pension, in order to improve the sustainability of the system, the government has the option of reducing the benefits to the elderly or increasing the burden on the working generation. Obtaining agreement on reform by both generations is often too difficult for the government to achieve. In this situation, some developed countries such as France have proceeded with the expansion of child benefit programs. These programs are expected to increase the population of the younger generation as a tax resource and, as result, to reduce the per capita fiscal burden in the future.

In this paper we analyze the relationship between child benefit and the fiscal burden in the setting of an overlapping generations (OLG) model. In the process of this analysis, it is important for us to distinguish between an exogenous fertility model and an endogenous fertility model. The reason is that recent studies have clarified that the Pareto-efficiency condition of the exogenous fertility model differs from that of the endogenous fertility model. First, for an exogenous fertility model, we use the OLG model introduced by Diamond (1965). Three types of steady states exist in the model: under-accumulation, golden rule, and over-accumulation. The first two steady states are Pareto-efficient, but the third is not. In addition, an empirical study by Abel, Mankiw, Summers, and Zeckhauser (1989) reports that in industrialized countries dynamic efficiency is satisfied. In a steady state, dynamic efficiency corresponds to under-accumulation (or golden rule). Therefore, the possibility that developed countries are in a state of under-accumulation seems high. In an exogenous fertility setting, an allocation is said to be Pareto-efficient if it is impossible to make some individuals better off without making other individuals worse off. For this reason, in an exogenous case, we cannot improve any generation's utility while at the same time sacrificing another generation's utility.

However, recent studies clarify the properties of the competitive equilibrium with an endogenous fertility setting. Raut and Srinivasan (1994) and Charkrabarti (1999) analyze the properties of intertemporal equilibrium with endogenous fertility. Conde-Ruiz et al. (2002) and Golosov et al. (2004) present the definition of Pareto-efficiency criteria in an endogenous fertility framework.

As a development of these studies, Michel and Wigniolle (2007)<sup>2</sup> point out the possibility that under-accumulation may not be efficient in an endogenous fertility setting. This implies that some

---

<sup>2</sup> Although there have been several approaches that endogenize fertility decisions, Michel and Wigniolle (2007) depend on the benchmark framework, which assumes that children are consumption goods that appear in the utility function of the parents. The basic articles are Becker (1960), Willis (1973), and Eckstein and Wolpin (1985). Other approaches depend on the literature based on the additional assumption of descendant altruism, as in Becker and Barro (1988) or the assumption of ascendant altruism and strategic behavior of parents, as in Nishimura and Zhang (1992).

policies could effect improvement in one generation's welfare without sacrificing another generation's welfare, even when it is in an under-accumulation state near the steady state. Moreover, the remarkable point made by Michel and Wigniolle (2007) is to clarify that the Representative-Consumer efficient (RC-efficient) condition, which is a concept developed in their study, has a profound connection with the sign-of-inequality relationship between the child-rearing cost and wage rate.<sup>3</sup> That is, if some policies do provide effects to this relationship, improvement in RC-efficiency becomes possible. Michel and Wigniolle (2007) provide proof that, by utilizing an OLG model with endogenous population growth, the possibility to improve RC-efficiency also exists in the case of under-accumulation. But they did not analyze an economy model with public debt. Therefore, Oguro and Takahata (2009) analyze the relationship between child benefit and fiscal burden, in the setting of an OLG model with both endogenous fertility and public debt, and provide the condition of RC-efficiency. However, they also could not analyze the relationship between child benefit and fiscal burden in a real economy. The reason is that the overlapping generations of their OLG model amount to only two: the working generation and the retired generation. To analyze the relationship in a real economy, it is necessary to build an OLG model with more overlapping generations: e.g., a model with 65 overlapping generations and endogenous fertility.

To this end, we construct a large-scale numerical dynamic equilibrium OLG model with endogenous fertility, which is calibrated to the Japanese economy. And we quantitatively evaluate the effects of child benefit change: e.g., the effects on the welfare of multiple generations. By doing this, we attempt to answer whether a fundamental change in child benefit policy results in significant positive effects on the Japanese economy, especially in terms of the government fiscal situation.

The structure of this paper is as follows. In Section 2, we describe the model structure; Section 3 presents the calibration strategy and the findings; Section 4 describes simulation results, and Section 5 contains concluding remarks and policy implications.

## 2. The Model Structure

In this section, we describe the demographic and economic structure of our model. The model used here is a computable general-equilibrium OLG model with perfect foresight agents, multiple periods, and endogenous fertility. In our model, there is a representative individual for each generation in the households sector. Each individual at age 20 maximizes his/her intertemporal utility function with consumption and number of children. The representative competitive firm has a standard Cobb-Douglas production technology and maximizes its profits. In our model, not only the goods market but also factor markets are perfectly competitive. The model has five main building blocks: 1) household behavior, 2) firm behavior, 3) the Government, 4) the public pension, and 5) market equilibrium. Details of each block follow.

---

<sup>3</sup> The definition of "RC-efficiency" can be seen in Michel and Wigniolle (2007) or Oguro and Takahata (2009).

(1) Household behavior

There is a representative individual for each generation in the household sector. We assume that preferences forms are the same for all agents in all generations. Moreover, each individual lives for a fixed number of periods. In each period of the model, the oldest generation dies and a new one enters. And the representative individuals maximize their intertemporal utility function with consumption and number of their children subject to their lifetime income. They are also assumed to be rational, having perfect foresight. Each generation enters the labor market at age 21, bears and brings up their children at ages 21 to  $M + 20$ , retires at age  $Q - 1$ , is granted a pension at  $Q$ , and dies at age  $Z$ . In addition, each supplies labor inelastically. The within-period utility function exhibits constant relative risk aversion, and preferences are additive and separable over time. In each region, the utility functions of the  $t$ th generation born in year  $t$  are specified as:<sup>4</sup>

$$U_t = \alpha \frac{n_t^{1-\sigma_1}}{1-\sigma_1} + (1-\alpha) \sum_{j=1}^{Z-20} \left( \frac{1}{1+\rho} \right)^j \frac{c_{t,j}^{1-\sigma_2}}{1-\sigma_2} \quad (1)$$

where  $\alpha$  refers to the weight between number of children and consumption,  $\sigma_1$  the preference parameter of number of children,  $j$  the  $j$ th period of life,  $\rho$  the pure rate of time preference, and  $\sigma_2$  the reverse of the elasticity of intertemporal substitution of consumption. The arguments of the utility function are the number of children ( $n_t$ ) and the consumption per period ( $c_{t,j}$ ). Leisure does not enter the utility function since the individual's labor supply is assumed to be exogenous.

In addition, we assume that the number of children ( $n_{t,j}$ ) whom the  $t$ th generation bears at the  $j$ th period of life is the following:

$$n_{t,j} = p_j n_t \quad (\text{where } p_j > 0 \text{ (if } 1 \leq j \leq M-20) \text{ and } p_j = 0 \text{ (if } j > M-20)) \quad (2)$$

where  $p_j$  refers to the possibility that each generation bears the children at the  $j$ th period of life and this parameter is assumed to be exogenous.

Moreover, the technological progress  $\lambda$  is assumed to be exogenous and labor embodied. We model age-specific labor productivity by assuming a hump-shaped age-earnings profile, i.e., a quadratic form of its age  $j$ , so its age-wage profile  $e_j$  takes the following form:

$$e_j = \zeta_0 + \zeta_1 j + \zeta_2 j^2, \quad \zeta_0, \zeta_1 \geq 0 \text{ and } \zeta_2 \leq 0 \quad (3)$$

The intertemporal budget equation of each generation is described as follows:

$$\sum_{j=1}^{M-20} \sum_{g=1}^{20} \frac{\phi_g (1-\theta_{t+j-1+g+20}) n_{t,j}}{\prod_{k=1}^{j-1+g} (1-tr_{t+k+20}) R_{t+k+20}} + \sum_{j=1}^{Z-20} \frac{(1+tc_{t+j+20}) c_{t,j}}{\prod_{k=1}^j (1-tr_{t+k+20}) R_{t+k+20}} = NW_t$$

---

<sup>4</sup> This is the expansion of the utility function provided by van Groezen, B., Leers, T., and Meijdam, L. (2003). If  $\sigma_1 = \sigma_2 = 1$  and  $Z=22$ , i.e. only two periods (working period and retired period), this utility function becomes the same form as that of van Groezen, B. et al. (2003).

$$\equiv \sum_{j=1}^{Q-21} \frac{(1-tw_{t+j+20} - \tau w_{t+j+20})w_{t+j+20}(1+\lambda)^{t+j+20}e_j}{\prod_{k=1}^j (1-tr_{t+k+20})R_{t+k+20}} + \sum_{j=Q-20}^{Z-20} \frac{(1-\tau p_{t+j+20})q_{t+j+20}}{\prod_{k=1}^j (1-tr_{t+k+20})R_{t+k+20}} \quad (4)$$

where  $1/\prod(1-tr)R$  refers to the factor of the present discounted value which is driven from the gross interest rate  $R_t$  and the capital tax  $tr_t$  in year  $t$ ,  $\phi_g$  is the child rearing cost at the  $g$  th period of life,  $\theta_t$  is the government subsidy in year  $t$ ,  $tc_t$  is the consumption tax rate in year  $t$ ,  $tw_t$  is the labor income tax rate in year  $t$ ,  $\tau w_t$  is the public pension contribution rate in year  $t$ ,  $NW_t$  is the net lifetime income of generation  $t$ ,  $w_t$  is the wage rate in year  $t$ ,  $\tau p_t$  is the tax for pension benefit in year  $t$ , and  $q_t$  stands for pension benefit in year  $t$ . In addition, child rearing cost is assumed to be proportional to net lifetime income, i.e.,  $\phi_g = \Phi_g NW_t$ , where  $\Phi_g$  is the constant parameter.

Each generation maximizes its utility function (1) under the budget constraint (4).

When  $\sigma \equiv \sigma_1 = \sigma_2$ , the maximization procedure differentiating the household utility function (2) with respect to  $n_t$  and  $c_{t,j}$ , subject to the individual's lifetime budget constraint (4), yields the following equations concerning consumption per period and number of children.

$$c_{t,j} = \left\{ \frac{(1-\alpha) \left( \frac{1}{1+\rho} \right)^j}{\mu \frac{(1+tc_{t+j+20})}{\prod_{k=1}^j (1-tr_{t+k+20})R_{t+k+20}}} \right\}^{1/\sigma}, \quad n_t = \left\{ \frac{\alpha}{\mu \sum_{j=1}^{M-20} \sum_{g=1}^{20} \frac{\phi_g (1-\theta_{t+j-1+g+20}) p_j}{\prod_{k=1}^{j-1+g} (1-tr_{t+k+20})R_{t+k+20}}} \right\}^{1/\sigma} \quad (5)$$

$$\text{, and } \mu^{1/\sigma} = \left[ \frac{\alpha^{1/\sigma}}{\left\{ \sum_{j=1}^{M-20} \sum_{g=1}^{20} \frac{\phi_g (1-\theta_{t+j-1+g+20}) p_j}{\prod_{k=1}^{j-1+g} (1-tr_{t+k+20})R_{t+k+20}} \right\}^{1/\sigma-1}} + \sum_{j=1}^{Z-20} \frac{(1-\alpha) \left( \frac{1}{1+\rho} \right)^{j/\sigma}}{\left\{ \frac{(1+tc_{t+j+20})}{\prod_{k=1}^j (1-tr_{t+k+20})R_{t+k+20}} \right\}^{1/\sigma-1}} \right] / NW_t$$

If the parameter  $\mu$  is stable, these equations dictate the following two relationships: 1) as in any life-cycle model, the trade-off between current and future consumption is determined by the ratio of the interest rate and the time preference rate, and by the degree of risk aversion, and 2) the number of children declines, when the child rearing cost increases or the government subsidy decreases. Moreover, from these equations, the following forms can be shown:

$$C_t = \sum_{j=1}^{Z-20} N_t c_{t-j-20,j}, \quad N_t = \sum_{j=1}^{M-20} N_t n_{t-j-20,j} \quad (6)$$

where  $C_t$  is the aggregated consumption in year  $t$ , and  $N_t$  measures the number of the generation born in year  $t$ . In addition, we can also derive the following physical wealth accumulation equation:

$$a_{t,j} = a_{t,j-1} (1-tr_{t+(j-1)+20})R_{t+(j-1)+20} + (1-tw_{t+j+20} - \tau w_{t+j-20})w_{t,j} - (1+tc_{t+j+20})c_{t,j} - \sum_{g=1}^j \phi_g (1-\theta_{t+j+20})n_{t,j-g+1}, \quad \text{and } PA_t = \sum_{j=1}^{Z-20} N_t a_{t-j-20,j} \quad (7)$$

where  $a_{t,j}$  is physical wealth asset of generation  $t$  at the  $j$  th period of life, and  $PA_t$  is the aggregated private asset in year  $t$ .

## (2) Firm behavior

The input/output structure is represented by the Cobb-Douglas production function with constant return to scale. The firm decides the demand for physical capital and effective labor in order to maximize its profit with the given factor prices of wage and rent, which are determined in the perfect competitive markets.

$$Y_t = AK_t^\varepsilon L_{e,t}^{1-\varepsilon}, \quad L_{e,t} \equiv \sum_{j=1}^{Q-20} (1+\lambda)^t e_j N_{t-j-20} \quad (8)$$

$$K_t = I_t + (1-\delta)K_{t-1} \quad (9)$$

where  $Y_t$  is output,  $\varepsilon$  stands for capital income share,  $A$  is a scale parameter,  $K_t$  is the physical capital stock, and  $L_{e,t}$  is the effective labor.

We can derive two factor prices, the rate of return  $r_t$  and the wage rate per unit of effective labor  $w_t$  by the first-order conditions for the firm's maximum profit:

$$R_t \equiv 1+r_t = \varepsilon AK_t^{\varepsilon-1} L_{e,t}^{1-\varepsilon} + (1-\delta), \quad w_t = (1-\varepsilon)AK_t^\varepsilon L_{e,t}^{-\varepsilon} \quad (10)$$

where  $\delta$  is the depreciation of physical capital.

## (3) The public pension

The pension sector grants a pension to the retirement generations while pension contribution is collected from the working generations.

$$P_t = \tau w_t \times w_t L_{e,t} \quad (11)$$

where  $P_t$  stands for the aggregated pension contribution.

The aggregated pension benefits in year  $t$  is given by the product of the population of retirement age, replacement rate, and average earnings of each generation during the working period  $\bar{W}_t$ .

$$B_t = \sum_{j=Q-19}^{Z-20} q_{t+j+20} N_{t+j+20} = \sum_{j=Q-19}^{Z-20} \gamma \times \bar{W}_{t+j+20} N_{t+j+20} \quad (12)$$

where  $\gamma$  denotes replacement rate and  $B_t$  is the aggregated pension benefit.

We explicitly model the public pension system as pay-as-you-go. The budget constraint of the pension sector can be shown as follows:

$$P_t = (1-sp)B_t \quad (13)$$

where  $sp$  denotes public subsidy to pension, which is financed by government expenditure  $G_t$ .

Moreover, we assume that the public pension sector maintains a fixed replacement rate exogenously. As a result, in our model, the pension contribution rate is endogenously determined in order to keep the budget constraint (13).

## (4) The Government

The government sector has four types of taxes: wage tax, consumption tax, capital tax, and pension benefit tax, and the public debt issue income as its revenue and pays the consumption, investment, and interest payments as expenditures.

$$T_t = tw_t \times w_t L_{e,t} + tc_t \times C_t + tr_t \times R_t \times PA_t + tp_t \times B_t \quad (14)$$

We keep all tax rates constant. The role of the government is to endogenously determine the rate of the public debt issue as a residual of government expenditure and revenue.

$$D_t = G_t - T_t + (1 + r_t)D_{t-1} \quad (15)$$

where  $G_t$  stands for government expenditure in year  $t$ ,  $T_t$  denotes tax revenue in year  $t$ ,  $D_t$  denotes public debt in year  $t$ .

The public debt issue  $Bond_t \equiv D_t - (1 + r_t)D_{t-1}$  is set endogenously due to the difference between expenditure and tax revenue. It should be noted that the public debt issue to GDP ratio will change over time as a result of possible imbalances between revenues and expenditures. Thus we don't know whether the fiscal policy of a country is sustainable and whether the government's intertemporal budget constraint must be satisfied.

## (5) Market equilibrium

Finally, in our model of a closed economy, we require the equilibrium in the financial market, i.e., the aggregate value of assets equals the market value of the capital stocks plus the value of outstanding government bonds:

$$PA_t = K_t + D_t \quad (16)$$

## 3. The Data, Calibration, and Scenarios

### 3.1 Data and calibration

First, we present the values of the main parameters and exogenous variables of the model in Table 1. The parameter values for the households' and firms' behaviors are derived from Auerbach and Kotlikoff (1987) and various early OLG simulation studies in Japan.<sup>5</sup> These parameters, such as technological and preference parameters except the weight parameter  $\alpha$ , are assumed to be constant.

The exogenous variables such as the macroeconomic, fiscal, and public pension variables are derived mainly from OECD (2007) "Tax Database," and Whitehouse (2007) "Pensions Panorama."

In addition, the child bearing possibility parameter is derived from the data of "Age-specific fertility rate," which is provided by the National Institute of Population and Social Security Research (2007), and the parameter values of the child rearing cost and the government subsidy are derived from the special research report about social cost of rearing children, which is provided by the Cabinet Office Director-General for Policies on a Cohesive Society, Japan (2005).

---

<sup>5</sup> See Sadahiro and Shimasawa (2001, 2003); Uemura (2002); and Ihori et al. (2006).



Second, by controlling the weight parameters in years 1900–2007, we calibrate our demographic projection to fit the data’s trend in “Population by Age (generation born in 1900–2007),” which is provided by the Statistics Bureau, Ministry of Internal Affairs and Communications with the collaboration of other Ministries and Agencies in Japan. Figure 1 reports the actual values and the computed values of demographic projection. Note that actual and calculated values correspond closely.

Next, in order to analyze the relationship between child benefit and fiscal burden, we start our calculations with a phase-in period of about 200 years in order to relax the unrealistic assumption of a steady state in the 2007 base year of our simulation. Moreover, since the model is simulated over 500 periods, we ensure a sufficiently long period for a steady state to be achieved.

Table 2 reports the actual values of some key variables in 2007 and the computed values in the model. Also, note that actual and calculated values correspond closely.

### 3.2. Scenarios

Next we present simulation scenarios. The scenarios are classified into four categories. Scenario 1 assumes the baseline case with no expansion of child benefit, and Scenarios 2 and 3 assume 100% increase of child benefit after 2015. Scenario 4 assumes 50% increase of child benefit after 2015. Scenarios 5 and 6 assume no expansion of child benefit but an increase in the consumption tax to 10% and 15% (consumption tax reform), respectively. Finally, Scenario 7 is the policy-mix of Scenario 2 (permanent expansion) and Scenario 6 (15% consumption tax reform).

In Scenarios 2 and 4, the increase of child benefit is permanent from 2015. In Scenario 3, the increase of child benefit is temporal for 2015–2025.

	Scenario 1	Scenario 2	Scenario 3	Scenario 4	Scenario 5	Scenario 6	Scenario 7
Child benefit	No increase	100% increase after 2015	100% increase for 2015-25	50% increase after 2015	No increase	No increase	100% increase after 2015
Consumption tax	5%	5%	5%	5%	10%	15%	15%

## 4. Simulation results

We now turn to describe the simulation results reported in Figures 2 to 5 and Table 3. Here we present the scenarios of results of the child benefit expansion in comparison to the cases of no expansion, the case of consumption tax reform, and the case of policy-mix (permanent expansion and consumption tax reform).

### (1) Demographic projection and macroeconomic variables

Figure 2 shows the population projection of future generations born in 2000–2050. The

projection of Scenario 1 and official estimation, which is provided by the National Institute of Population and Social Security Research (2006), closely correspond. In Scenario 2 (100% permanent child benefit increase), compared with Scenario 1, the population of the generation born in 2030 increases by 143,000, in Scenario 3 (100% temporally child benefit increase) by 11,000, in Scenario 4 (50% permanent child benefit increase) by 66,000, in Scenario 5 (10% consumption tax reform) by -4,000, in Scenario 6 (15% consumption tax reform) by 19,000, and in Scenario 7 (policy-mix) by 166,000.<sup>6</sup> However, in Scenario 6, the increase of the future generation is temporal. In this scenario compared with Scenario 1, the population of the generation born in 2050 decreases by 83,000.

Figure 3 also shows the retired population ratio. The ratio of Scenario 1 and official estimation, which is provided by the National Institute of Population and Social Security Research (2006), closely correspond. In Scenario 2, compared with Scenario 1, the ratio in 2050 decreases by 1.7%, in Scenario 3 by 0.1%, and in Scenario 4 by 0.8%. Thus it can be seen that these child benefit expansions slightly decrease the progress of population aging.

As we adopt the lifecycle hypothesis, the saving rate is severely affected by the rise of the rate of elderly population, which is strongly correlated with the demographic trend. In Scenarios 1 to 7, there is no significant change in the trend of the saving rate during the simulation periods. But its level differs in each scenario. In Scenario 1, the saving rate shows a tendency to decrease from 6.17% in 2007 to -11.75% in 2050. Table 3 shows that the child benefit expansions basically raise the saving rate in 2007 to 2040 years. On the other hand, the reform with higher consumption tax reduces the saving rate more in the years from 2007 to 2040.

Because of the assumed technology and lifecycle hypothesis, the GDP is determined mainly by working-age population dynamics. In the baseline scenario, the GDP level grows stagnant. It declines markedly from 2020 to 2050, reflecting the declining labor force. And then, in each scenario, the GDP declines to 53.79%–58.73% in 2050 from the base year 2007. But, in Scenarios 1 to 5, GDP per employee increases from 2007 to 2050. On the other hand, in Scenarios 6 and 7, GDP per employee temporally decreases in 2015 and increases after 2020.

Finally, we briefly evaluate factor prices. In each scenario, due to the capital market equilibrium, the interest rate (wage rate) fluctuates within a narrow range, e.g., 2.61%–3.89% (93.55%–101.65%) over five decades.

## (2) Fiscal variables

Generally, the child benefit expansion can be expected to give the fiscal balance ambivalent effects through several channels. If the expansion is financed by new public bond issues, it initially increases public debt. But the increase in the number of children also increases tax bases, and then

---

<sup>6</sup> Consumption tax reform may contribute to the increase of population of future generations through the mechanism in that the fall in net lifetime income decreases the child rearing cost, e.g., the opportunity cost which is the net lost income when parents bring up a child.

changes the trend of government revenue and expenditure. As a result, the future government debt will be either reduced or increased.

Table 3 shows that in Scenario 2, compared with Scenario 1, the Debt-GDP ratio slightly increases by 6.99% in 2030 and 13.18% in 2050. In Scenario 3, the ratio increases by 5.05% in 2030 and 13.41% in 2050, and in Scenario 4 by 3.30% and 6.29% in 2050. Figure 4 also shows that in Scenario 2 compared with Scenario 1, the debt per employee slightly decreases by 0.04 in 2050 and in Scenario 4 by 0.02 in 2050. However, in Scenario 3, the debt per employee increases by 0.05.

On the other hand, in Scenarios 5 to 7 (consumption tax reform or policy-mix), the Debt-GDP ratio is reduced by 58.80%–113.23% in 2030 and 326.87%–587.58% in 2050, and the debt per employee is reduced by 1.32–2.36 in 2050.

### (3) Welfare

Figure 5 shows generational welfares of Scenarios 1 to 7. These are welfares of subsequent cohorts measured in terms of lifetime utility level against the cohort born in 1930. The long-run increase in the pension premium to wage rate caused by the progress of aging decreases the amount of resources available within their lifetime. The long-run increase in the public debt to GDP ratio also reduces private capital stock available and possibly decreases future growth. Current and future generations suffer a severe welfare loss.

In Scenarios 1 to 7 in Figure 5, we measure the welfare of each generation with equivalent variation. The welfares of Scenario 6 gradually decline and the bottom for this scenario doesn't occur until birth year 2050, but Scenarios 1 to 4 have a welfare bottom at the generation born in 2025; the bottom for Scenario 5 is in 2030, and for Scenario 7 it is in 1990.

In addition, in Scenarios 2 to 4, compared with Scenario 1, all generations born after 1990 obtain a welfare gain: e.g., in Scenario 2, the welfare of the generation born in 2030 dramatically increases by 4.6%, in Scenario 3 by 0.2%, and in Scenario 4 by 2.2%. This means that the welfare conditions in Scenarios 2 to 4 correspond to the concept of “RC-improvement” developed by Michel and Wigniolle (2007).

On the other hand, in Scenarios 5 to 7, compared with Scenario 1, most generations born after 1940 suffer a welfare loss whose burden is covered by an increase in consumption tax. However, if Scenario 6 is an inevitable choice in order to maintain the sustainability of fiscal budget, we should change the baseline scenario from Scenario 1 to Scenario 6. Then, in Scenario 7, compared with Scenario 6, all generations born after 1990 obtain a welfare gain. This means that Scenario 7 also becomes an “RC-improvement.”

Therefore, from the comparison between the child benefit scenarios, the consumption tax reform scenarios, and the policy-mix scenario, we draw the following conclusions: 1) if we can ignore the sustainability of the fiscal budget in Scenarios 2 to 4, the child benefit expansions are

expected to make some contributions to the improvement of the welfare of current and future generations, and 2) if both the sustainability of the fiscal budget and the improvement of the welfare of current and future generations are requirements, we will need to promote a policy such as a policy-mix with the child benefit expansion and additional fiscal reform, i.e. increasing the consumption tax. Then, the policy-mix can be expected to provide a higher level of welfare for current and future generations than only consumption tax reform.

## 5. Concluding remarks

In this paper, we presented an OLG simulation model with endogenous fertility in order to analyze the relationship between child benefit and fiscal burden in Japan. Our simulation results show that expansion of the child benefit will improve the welfare of current and future generations. On the other hand, our findings show that we cannot expect a significant long-term improvement in welfare solely from implementing a policy of increasing the consumption tax. If both the sustainability of the fiscal budget and the improvement of the welfare of current and future generations are requirements, we will need to promote a strategy consisting of such components as a policy-mix that includes both child benefit expansion and additional fiscal reform, i.e. increasing the consumption tax. Implementation of such a the policy-mix can be expected to provide a higher economic level in the welfare of current and future generations could be expected solely from consumption tax reform.

The weakness of our study is that our model does not include the following points: 1) the effect of heterogeneous households with different preferences such as a child bearing parameter, 2) the effect of endogenous labor supply, 3) the effect of not only the child as consumption but also that as investment, e.g., child for “old-age annuity,” and 4) the effect of the global capital market. These points remain subjects for future study.

In addition, the Japanese government is currently trying to develop a model for estimating the direction of the population of future generations, which has economic underpinnings. Therefore, if our model can be made more robust, it may prove to have a great impact on the method for estimation of the Japanese population, by which we analyze the relationship between future population and changes in the economic environment.

Finally, in an era of population aging, Japan will face enormous difficulties. Even given the difficulty of the task, Japan, like other developed countries, must confront these and other obstacles and solve the related issues to chart a productive and viable future for its future generations.

## References

- Abel, A. B., Mankiw, N. G., Summers, L. H., and Zeckhauser, R. J. (1989), “Assessing Dynamic Efficiency: Theory and Evidence,” *Review of Economics Studies* 56(1), pp. 1–20.
- Becker, G. S. (1960), “An economic analysis of fertility. In: Demographic and economic change in developed countries,” *National Bureau of Economic Research Conference Series* 11, pp. 209–231.
- Becker, G. S. and Barro, R. J. (1988), “A reformulation of the economic theory of fertility,” *Quarterly Journal Of Economics* 103, pp. 1–25.
- Chakrabarti, R. (1999), “Endogenous fertility and growth in a model with old age support,” *Economic Theory* 13, pp. 393–416.
- Conde-Ruiz, J. I. and Gimenez, E. L. (2002), “Perez-Nievas, M.: Efficiency in an overlapping generations model with endogenous population,” *mimeo*.
- Diamond, P.A. (1965), “National Debt in a Neoclassical Growth Model,” *American Economic Review* 55(5), pp. 1126–1150.
- Eckstein, Z. and Wolpin, K. (1985), “Endogenous fertility and optimal population size,” *Journal of Public Economics* 27, pp. 93–106.
- Golosov, M., Jones, L. E. and Tertilt, M. (2004), “Efficiency with endogenous population growth,” *National Bureau of Economic Research* 10231.
- Ihori, T., Kato, R., Kawade, M. and Bessho, S. (2006). “Public debt and economic growth in an aging Japan,” K. Kaizuka and Ann O. Krueger, eds., *Tackling Japan's Fiscal Challenges: Strategies to Cope with High Public Debt and Population Aging*. [in Japanese]
- Michel, P. and Wigniolle, B. (2007), “On Efficient Child Making,” *Economic Theory* 31(2), pp. 307–326.
- National Institute of Population and Social Security Research. *Population Statistics of Japan, 2007*. [in Japanese]
- \_\_\_\_\_. *Population Projections for Japan: 2006-2055*, 2006.
- Nishimura, K. and Zhang, J. (1992), “Pay-as-you-go public pensions with endogenous fertility,” *Journal of Public Economics* 48, pp. 239–258.
- OECD. *Tax Database*, Paris, 2007.
- Oguro, K. and Takahata, J. (2009), “Child Benefit and Fiscal Burden with Endogenous Fertility,” Public Economics Program, School of International and Public Policy, Hitotsubashi University, Discussion Papers on Demography and Policy #2009-001.
- Raut, L. K. and Srinivasan, T. N. (1994), “Dynamics of endogenous growth,” *Economic Theory* 4, pp. 777–790.
- Sadahiro, A., and M. Shimasawa (2001). “Fiscal Sustainability and the Primary Surplus: A Simulation Analysis with OLG Model,” *JCER Economic Journal* 43. pp. 117–132. [in Japanese]
- \_\_\_\_\_ and \_\_\_\_\_ (2003). “The computable overlapping generations model with an

endogenous growth mechanism,” *Economic Modeling* 20(1), pp. 1–24.

- Uemura, T. (2002). “Lifecycle General Equilibrium Analysis on Social Security,” *Keizaironshu* 28(1). [in Japanese]
- van Groezen, B., Leers, T., and Meijdam, L. (2003) “Social Security and Endogenous Fertility: Pensions and Child Allowances as Siamese Twins,” *Journal of Public Economics* 87, pp. 233–251.
- Whitehouse, Edward. *Pensions Panorama: Retirement-Income Systems in 53 Countries*, The World Bank, 2007.
- Willis, R. (1973), “A new approach to the economic theory of fertility behavior,” *Journal of Political Economy* 81, S14–S64.

Table 1 Parameter Values of the Model

PARAMETER		VALUE	
<i>Utility function</i>			
Time preference rate	$\rho$	0.01	
Intertemporal elasticity of substitution	$1/\sigma$	2.0	
Weight parameter between children number and consumption	$\alpha$	0.84*	
<i>Production function</i>			
Technology progress	$\lambda$	0.002	
Capital share in production	$\varepsilon$	0.3	
Physical capital depreciation	$\delta$	0.05	
<i>Tax policy parameters</i>			
Wage tax	$tw$	20.0%	
Capital tax	$tr$	20.0%	
Consumption tax	$tc$	5.0%	
Pension benefit tax	$tp$	10.0%	
<i>Pension policy parameters</i>			
National subsidy to pension	$sp$	25.0%	
Replacement ratio	$\gamma$	50.0%	
<i>Other parameters</i>			
Child rearing cost to net lifetime income	$\Phi_g$ g=	0 to 5	0.78%
		6 to 10	0.46%
		11 to 15	0.55%
		16 to 20	0.58%
Child bearing possibility	$\rho_j$ j=	1 to 5	3.0%
		6 to 10	7.4%
		11 to 15	7.0%
		16 to 20	2.6%
Government subsidy to child rearing cost	$\theta$	0.1	
Limit age of bearing child	$M$	40	
Age of retirement	$Q$	65	
Average life expectancy	$Z$	85	

\* This parameter is fixed after year 2007.

*Not to be quoted without express written permission from the authors*

Table 2 Year 2007 of the Baseline Scenario

	OFFICIAL	MODEL
<i>National Income (% of GDP)</i>		
Private consumption	74.1%	81.3%
Government purchases of goods and services	21.0%	24.3%
Saving rate	3.1%	6.1%
<i>Government Indicators</i>		
Pension premium to wage	14.9%	14.9%
Gross public debt ( <i>% of GDP</i> )	170.6%	170.6%
Primary balance ( <i>% of GDP</i> )	-2.4%	-4.5%
Tax revenues ( <i>% of GDP</i> )	18.4%	19.8%
<i>Other Indicators</i>		
Capital output ratio	2.9	4.6
Interest rate	1.7%	2.6%

Data source: Official values are derived from OECD Economic Outlook No. 84, 2008 and "Annual Report on National Accounts," the Japanese SNA statistics (Cabinet Office).



*Not to be quoted without express written permission from the authors*

Table 3 Simulation Results— Macro Economic Projection

		GDP	GDP per employee	Saving rate	Capital-labor ratio	Interest rate	Wage rate	Debt-GDP ratio	Debt per employee	Pension premium to wage
Scenario 1	2007	100.00%	100.00%	6.17%	100.00%	2.61%	100.00%	170.65%	100.00%	14.90%
	2010	98.63%	101.30%	5.59%	101.82%	2.51%	100.54%	186.94%	108.02%	15.83%
	2015	94.49%	104.03%	5.82%	104.68%	2.37%	101.38%	219.79%	123.21%	17.37%
	2020	90.34%	106.21%	0.69%	105.60%	2.32%	101.65%	259.60%	141.98%	25.99%
	2030	80.75%	103.92%	1.16%	95.61%	2.85%	98.66%	364.40%	192.97%	26.45%
	2040	66.59%	102.40%	3.45%	87.25%	3.37%	95.99%	559.78%	283.55%	28.90%
	2050	53.79%	102.58%	-11.75%	80.25%	3.87%	93.61%	936.46%	429.65%	37.35%
Scenario 2	2007	100.00%	100.00%	6.17%	100.00%	2.61%	100.00%	170.79%	100.00%	14.90%
	2010	98.63%	101.30%	5.59%	101.82%	2.51%	100.54%	187.15%	108.06%	15.83%
	2015	94.51%	104.05%	6.06%	104.74%	2.36%	101.40%	220.38%	123.46%	17.37%
	2020	90.33%	106.20%	0.95%	105.57%	2.32%	101.64%	261.87%	142.96%	25.99%
	2030	80.66%	103.81%	1.49%	95.27%	2.87%	98.56%	371.39%	194.48%	26.48%
	2040	66.39%	101.88%	3.99%	85.96%	3.46%	95.56%	576.67%	283.88%	29.02%
	2050	54.84%	102.01%	-10.62%	80.07%	3.89%	93.55%	949.64%	425.09%	36.77%
Scenario 3	2007	100.00%	100.00%	6.17%	100.00%	2.61%	100.00%	170.77%	100.00%	14.90%
	2010	98.63%	101.30%	5.59%	101.82%	2.51%	100.54%	187.12%	108.05%	15.83%
	2015	94.49%	104.03%	6.05%	104.69%	2.37%	101.39%	220.33%	123.43%	17.37%
	2020	90.31%	106.18%	0.93%	105.52%	2.33%	101.63%	261.71%	142.97%	25.99%
	2030	80.67%	103.82%	1.24%	95.30%	2.87%	98.57%	369.45%	195.11%	26.48%
	2040	66.40%	102.08%	3.58%	86.37%	3.43%	95.70%	570.02%	287.17%	29.01%
	2050	53.95%	102.60%	-12.06%	80.44%	3.86%	93.68%	949.87%	435.66%	37.34%

*Not to be quoted without express written permission from the authors*

Table 3 Simulation Results— Macro Economic Projection (continued)

		GDP	GDP per employee	Saving rate	Capital- labor ratio	Interest rate	Wage rate	Debt-GDP ratio	Debt per employee	Pension premium to wage
Scenario 4	2007	100.00%	100.00%	6.17%	100.00%	2.61%	100.00%	170.72%	100.00%	14.90%
	2010	98.63%	101.30%	5.59%	101.82%	2.51%	100.54%	187.05%	108.04%	15.83%
	2015	94.50%	104.04%	5.94%	104.71%	2.37%	101.39%	220.08%	123.33%	17.37%
	2020	90.33%	106.20%	0.81%	105.58%	2.32%	101.64%	260.70%	142.45%	25.99%
	2030	80.71%	103.87%	1.31%	95.45%	2.86%	98.61%	367.70%	193.71%	26.46%
	2040	66.49%	102.16%	3.70%	86.64%	3.41%	95.79%	567.67%	283.75%	28.95%
	2050	54.27%	102.31%	-11.23%	80.17%	3.88%	93.58%	942.75%	427.62%	37.08%
Scenario 5	2007	100.00%	100.00%	6.17%	100.00%	2.61%	100.00%	170.49%	100.00%	14.90%
	2010	98.63%	101.30%	5.59%	101.82%	2.51%	100.54%	186.71%	107.98%	15.83%
	2015	92.39%	101.72%	6.86%	97.14%	2.76%	99.13%	222.73%	122.20%	17.37%
	2020	89.40%	105.10%	-0.43%	101.98%	2.50%	100.59%	246.94%	133.75%	26.38%
	2030	81.08%	104.35%	-2.02%	96.93%	2.78%	99.07%	305.60%	162.55%	26.65%
	2040	68.73%	105.66%	-0.12%	96.88%	2.78%	99.05%	409.28%	214.58%	28.30%
	2050	54.73%	104.22%	-2.92%	84.65%	3.55%	95.12%	609.59%	286.57%	36.02%
Scenario 6	2007	100.00%	100.00%	6.17%	100.00%	2.61%	100.00%	170.38%	100.00%	14.90%
	2010	98.63%	101.30%	5.59%	101.82%	2.51%	100.54%	186.51%	107.95%	15.83%
	2015	90.47%	99.60%	7.84%	90.56%	3.15%	97.07%	225.78%	121.38%	17.37%
	2020	88.56%	104.12%	-1.65%	98.84%	2.67%	99.65%	235.22%	126.26%	26.74%
	2030	81.28%	104.60%	-5.45%	97.71%	2.73%	99.31%	251.16%	133.71%	26.89%
	2040	69.67%	107.02%	-4.70%	101.18%	2.55%	100.35%	285.97%	151.73%	28.26%
	2050	57.90%	109.58%	-6.85%	100.40%	2.59%	100.12%	348.88%	174.13%	33.96%

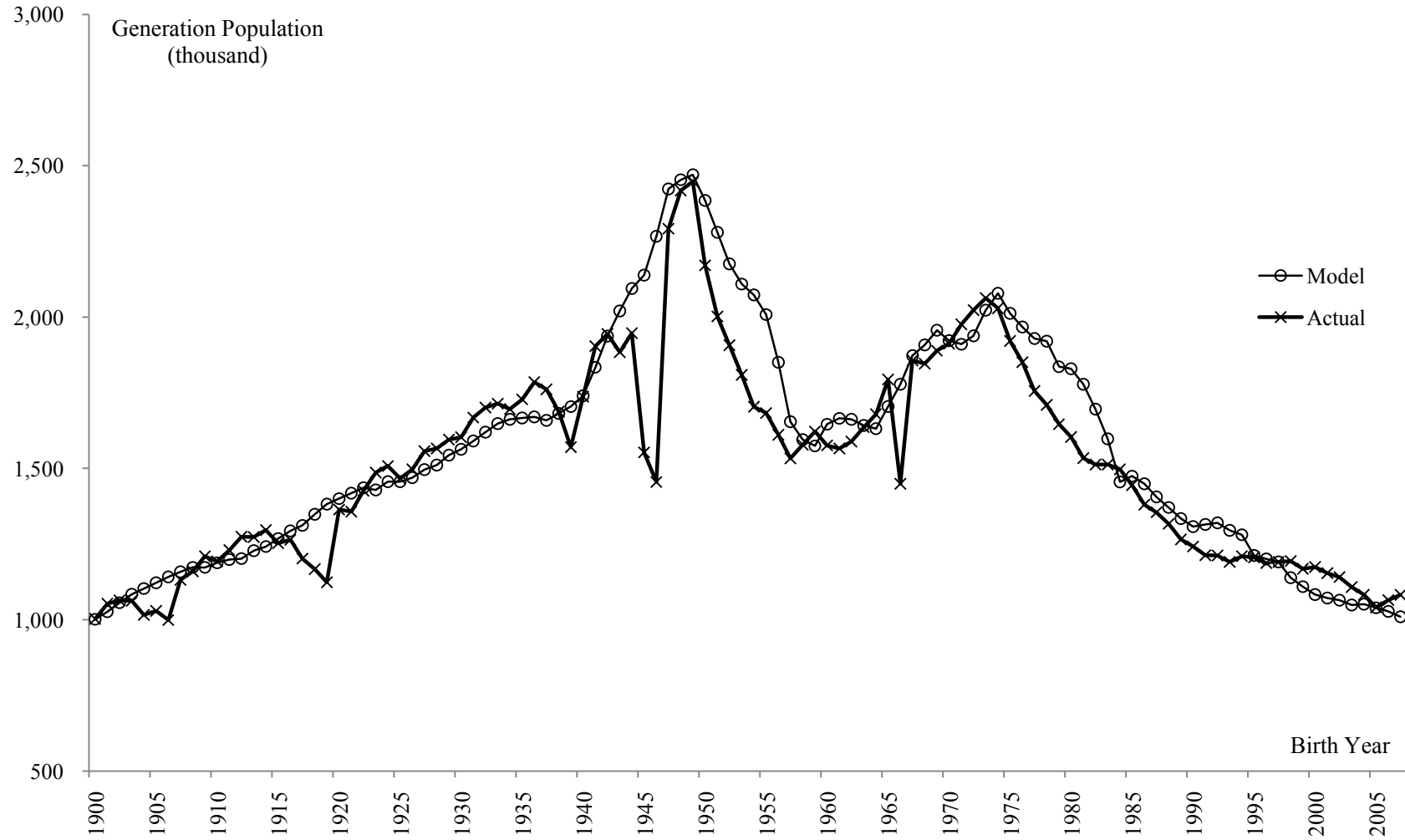
*Not to be quoted without express written permission from the authors*

Table 3 Simulation Results— Macro Economic Projection (continued)

		GDP	GDP per employee	Saving rate	Capital- labor ratio	Interest rate	Wage rate	Debt-GDP ratio	Debt per employee	Pension premium to wage
Scenario 7	2007	100.00%	100.00%	6.17%	100.00%	2.61%	100.00%	170.51%	100.00%	14.90%
	2010	98.63%	101.30%	5.59%	101.82%	2.51%	100.54%	186.73%	107.99%	15.83%
	2015	90.49%	99.63%	8.14%	90.63%	3.15%	97.09%	226.46%	121.67%	17.37%
	2020	88.55%	104.11%	-1.31%	98.80%	2.67%	99.64%	237.98%	127.50%	26.74%
	2030	81.19%	104.49%	-4.99%	97.37%	2.75%	99.20%	259.50%	136.54%	26.92%
	2040	69.61%	106.69%	-4.00%	100.36%	2.59%	100.11%	302.43%	156.29%	28.30%
	2050	58.73%	108.26%	-5.40%	98.04%	2.71%	99.41%	371.93%	180.39%	33.57%

*Not to be quoted without express written permission from the authors*

Figure 1 Demographic Projection of Each Generation



*Not to be quoted without express written permission from the authors*

Figure 2 Simulation Results—Demographic Projection of Future Generation

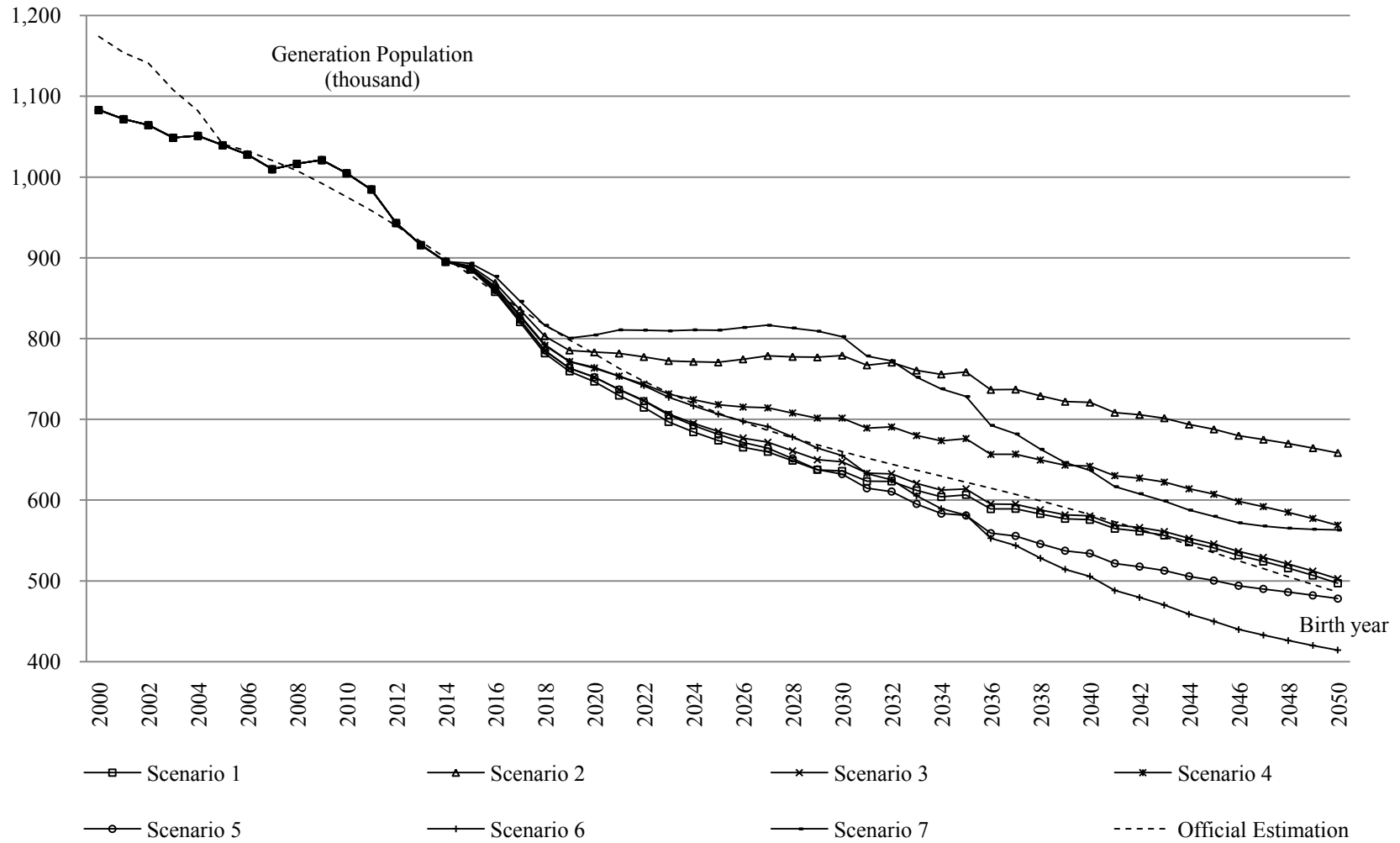


Figure 3 Simulation Results—Retired population ratio

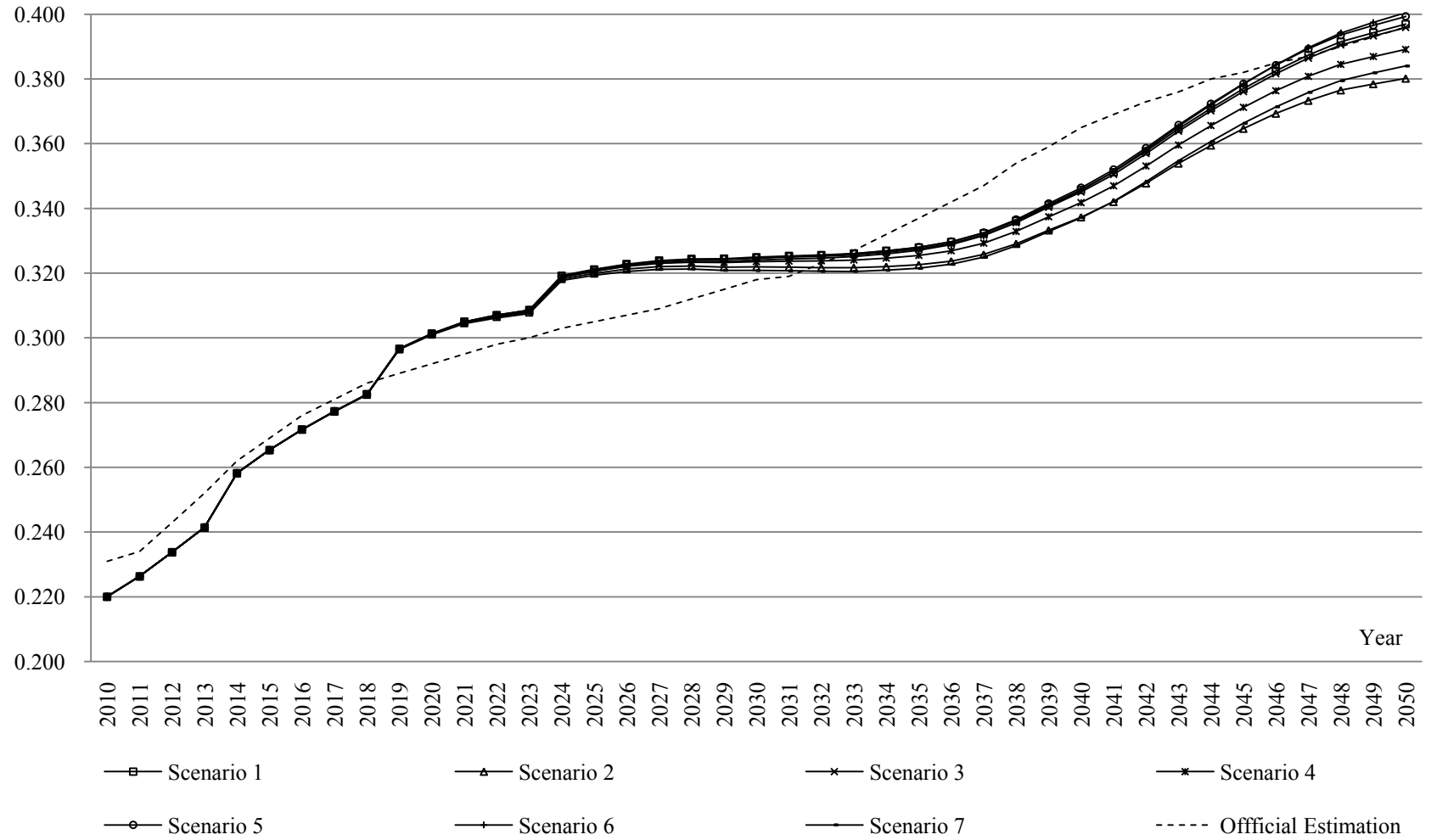
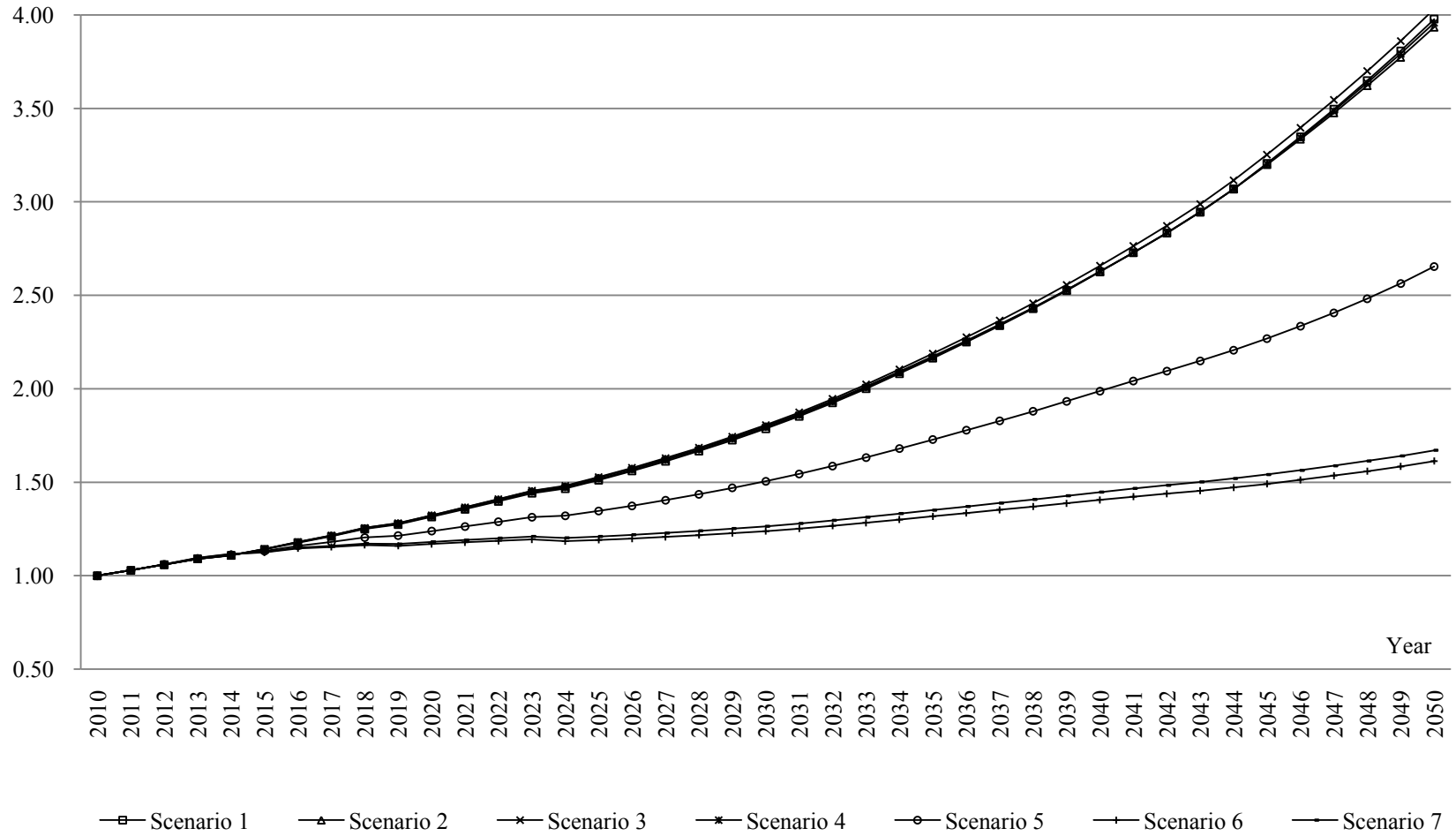


Figure 4 Simulation Results—Debt per employee



*Not to be quoted without express written permission from the authors*

Figure 5 Simulation Results – Welfare with Equivalent Variation

