Discussion Papers on Demography and Policy

#2009-004

School Education, Learning-by-doing and Fertility in Economic Development

Koji Kitaura
Gifu University

Akira Yakita
Nagoya City University

September 2009

Hitotsubashi University

Public Economics Program
School of International and Public Policy
Tokyo, Japan

School education, learning-by-doing and fertility in economic development

Koji Kitaura[a] and Akira Yakita[b*]

[a] Gifu University

[b] Nagoya City University

Abstract

This paper examines the policy implications of relaxing constraints on the educational choice of individuals for economic development. Distinguishing human capital accumulation through schooling from one through learning-by-doing and knowledge spillovers, we show that in earlier stages of development, mitigating and eventually eliminating constraints on school education would be necessary for even further economic development. Expanded school education increases the income of individuals and encourages physical capital accumulation, which enlarges productive knowledge through implementation and operations. The raised labor productivity thus boosts economic growth. In the process, the fertility rate will decline because of the increased education cost per child.

JEL classification: D91; I21; J13; O11

Keywords: School education; learning-by-doing and knowledge spillovers; economic development

* Corresponding author: yakita@econ.nagoya-cu.ac.jp (A .Yakita)

1. Introduction

Even at the elementary education level, more than 15% of children can not have an access to schools in developing economies, such Arab states and North Africa, from geographical, economic, social and cultural reasons (JICA, 2004: p. 2).[1] Besides the problem of maldistribution of higher education caused by income inequality within each developing economy, the shares of public education spending on secondary and tertiary education in developing countries are also lower than those in OECD countries, while the shares on primary education are rather higher than those in the developed economies (World Bank, 1995).[2] It is well known that higher education may play a crucial role in further economic development. The purpose of this paper is twofold: to analyze the role of human capital accumulation in economic development especially when the educational choices of individuals, especially on secondary and tertiary education, are constrained because of low incomes and, at the same time, there is poor productive knowledge which can be accumulated along with capital accumulation; and then to investigate the policy implications of relaxing constraints on economic development and fertility.

For these purposes, we distinguish two processes of human capital accumulation: one through school education, hereafter called academic knowledge in this study, and the other through learning-by-doing and knowledge spillovers in the production process, which is called productive knowledge. Productive knowledge obtained through learning-by-doing increases with new physical capital investment and/or capital accumulation, as suggested by Arrow (1962) and formalized by Romer (1986), while individuals' education investment through schooling may be subject to diminishing marginal returns, as supposed by, for example, Galor and Moav (2004).[3] However, it will be more rewarding for underdeveloped

---

[1] Sato (2004) pointed out that the ratio of population of age 0 to 15 is 56.4% on average for developing countries (28.5% for developed countries) in 1997, which strengthens the pressure on (public) educational expenditure in these countries.

[2] An exception is only the share of secondary education for three countries in Middle East and North Africa (see World Bank 1995; Table 3.2 on p. 56).

[3] The causality between school resources and school performance is still controversial. See, for example, Hanushek (1986), Barro and Lee (2000), Kruger and Lindahl (2001) and

economies to acquire already established knowledge through formal schooling rather than using the trial-and-error process, while advanced technology can be learned and absorbed, sometimes with modifications and/or improvements, when implementing and dealing with new machinery. The stock of human capital of a worker can be considered as the mixture of these two types of knowledge accumulated in different ways. In the literature, however, these two have not been taken into account distinctly and simultaneously in a single model of endogenous economic growth. This is the first and most important point of this paper. Although Galor and Moav (2004) recently modeled the shift of the growth engine from physical capital to human capital, we focus on productive knowledge brought about by physical capital accumulation rather than physical capital stock itself.[4]

Our model has two other salient features. First, we assume that there is some constraint on the educational choices of individuals in the earlier stages of development of economies. In this study, assuming implicitly that most children receive elementary, often compulsory, education, we are mainly concerned with secondary academic education through formal schooling. If school education services are provided collectively or by governments, the education level that individuals can enjoy will be constrained by the level provided in the society when the income levels of both individuals and the society as a whole are low. The finance of school facilities or institutes may often be characterized by scale indivisibilities or other types of non-convexity of the cost, so that schools can not be funded in low-income economies. One of the reasons for children who find it difficult to attend school is often suggested in the literature as insufficient number of schools (e.g., JICA, 2004; Sato, 2004). It is pointed out in World Bank (1995; p. 42) that there is strong evidence around the developing world that many 12-to-17-year-olds at the secondary education level are not in school because of a lack of places rather than a lack of interest.[5]

Lee and Barro (2001).

[4] Galor and Moav (2004) rather emphasized the role of income inequality during the transition process from the Industrial Revolution to modern growth.

[5] World Bank (1995) also suggested that "the gap between demand and supply at the secondary levels reflects population growth, the increasing proportion of students

We explicitly introduce such a constraint on the education level of individuals at earlier stages of economic development. The constraint contrasts to the borrowing constraint or liquidity constraint discussed in the literature (e.g., Galor and Zeira, 1993; de Gregorio, 1996). In later stages of development the constraint ceases to bind, and individuals will choose to enjoy any level of education they prefer.

Second, we assume that individuals choose not only the education levels of their offspring but also the number of children they will have. The trade-off between the quality and quantity of children has been analyzed, for example, by Becker and Lewis (1973), and, more recently, Moav (2005), and de la Croix and Doepke (2003) pointed out that the fertility choice of individuals plays an important role in economic development. The constraint on individuals' choices of their education level will also affect the trade-off. We thus analyze the effects of the educational constraint on fertility as well.

The results of this study are as follows. At earlier stages of economic development, as the educational constraint is relaxed, the accumulation of academic knowledge will be accelerated, and the income level and/or growth will become higher. Savings and hence physical capital accumulation will be stimulated through the income effects. This accelerated capital accumulation increases productive knowledge in production places, which raises labor productivity and thereby promotes economic growth. However, since a rise in the education level of children (due to the relaxation of the constraint) increases the education costs to parents, they will have fewer children. Therefore, during periods of relaxing educational constraints, the fertility rate continues to decline.[6]

Rises in the education level brought about by relaxing the educational constraint,

---

completing primary school, governments' difficulties in financing an expanded public system, poor parents' difficulties in paying school fees, and restrictions on private schooling" (p. 43).

[6] Galor and Weil (1999, 2000), authors of seminal works in this field, argued that the increase in the rate of technological progress gradually increased demand for human capital in the second phase of the Industrial Revolution, and that improved technology eased parents' budget constraints. But they did not consider the conditions of the supply side of human capital, i.e. schools and teachers, explicitly. For recent arguments on the so-called unified growth theories, see Galor (2005).

consequent increases in the growth rate and declines in the fertility rate not only seem to be consistent with the experience of Japan since the Meiji Resolution, but contrast with the experience of the pioneers of economic development. For example, in England, the acceleration of per capita GDP Growth and the number of average schooling years occurred almost at the same time in the nineteenth century (see Matthews et al. (1982) and Maddison (2001)). Our result has therefore important implications for development and education policies. It is crucial to relax any constraints on education and make them cease to bind in the earlier stages of economic development lest economic growth be hindered.

The remainder of this paper is organized as follows. In the next section we survey the development process since the Meiji Resolution in Japan, which is called a latecomer economy by Ohkawa and Kohama (1986). Section 3 introduces an overlapping generations model with human capital accumulation. In Section 4, we analyze the dynamics of the model and examine the effects of educational constraints on dynamic behaviors of the model. Then, in Section 5, we analyze the effects of relaxing constraints on growth and fertility. The conclusion is stated in the last section.

2. Experience of Japan

In analyzing Japan's development process, Ohkawa and Kohama (1986) conjectured that the long-term tendency of increase in the proportion of educational expenditure to gross domestic capital formation may be a characteristic of economically successful latecomers. In this section we examine Japan's experience of economic development and education policy since the Meiji period (1868-1912), in particular focusing on the period after the 1890's during which the school system in Japan is said to have been established (Ministry of Education of Japan, 1962). The education system of secondary schools (and universities including colleges) was expanded in 1918. The number of students attending the secondary schools had increased rapidly after about 1910, as depicted in Figure 1. Then, in 1947 after World War II, Japan drastically reformed their educational system. Junior high school education became compulsory, and the number of secondary schools increased correspondingly. The ratio of the number of children attending secondary-education-level schools to the total population of corresponding ages has increased largely over time. In particular, the ratio of children in secondary schools increased discontinuously because of the reform, i.e. from 46% in 1940 to 61.7% in 1947. During the 1947-1955 period, it nevertheless rapidly increased. The ratio of students attending senior high schools in the total population of corresponding ages (16-18 year old) was about 51% in 1955, although the speed of increase in the ratio was relatively small, reaching 70% in 1965 and about 92% in 1985 (although we show only the total number of students in junior and senior high schools and the corresponding age population in Figure 1).

Figure 1 also illustrates the number of schools and numbers of students per teacher in secondary schools for the 1890-1945 period and in senior high schools for the 1950-2005 period, respectively. Except for discontinuity between 1947 to 1950, the rate of change in the number of schools has definitely been smaller than that of the number of students attending school during the period around 1905-1965. As a consequence, the (average) number of students per school was expected to increase. Figure 1 shows that the number of students per teacher had an upward trend from 1910 to 1965, with discontinuity between 1947 and 1950. Even after 1965, the numbers of both schools and teachers increased but more slowly, while the population of ages 12 to 17 turned to be on a downward trend (although fluctuating largely). It should be noted that the number of teachers increased from 1960 to 1965, and that the number of students per teacher declined since 1965.

Kaser (1966) mentioned that Japan expanded its secondary education system at an early stage both in its economic development and education system compared to other (Western) industrialized countries. Indeed, the burden of education was considerably severe, at least for residents of each school district as a whole. In 1947, local and municipal governments started to build their junior high schools half with subsidies from the central government and half by issuing debts. However, because of the balanced budget policy of the central government starting in 1949, some schools had to be foregone and others partly financed, about 40 to 60 per cent, with contributions (which were actually compulsorily-assessed payments) by the residents, although the education system of 9-year compulsory education is said to be completed about in 1950 (see Ministry of Education of Japan (1954)). In this sense government policy was not consistent in the early stage of the new educational regime, and the resulting number of schools was smaller than initially scheduled. The situation might be similar for senior high schools. However, although the educational policy pushed by the government lacked sufficient financial support, people sought schools of higher levels to earn higher incomes in the future (e.g., Terao, 1956). Until about 1985, the numbers of schools and teachers continued to increase. Especially, the number of private senior high schools increased from 895 in 1955 to 1224 in 1970 (the ratios in the total number of senior high schools were 19.43% and 25.51%, respectively).[7] About three fourths of senior high schools were founded by national, local and municipal governments, and more than 70% of the students attended the publicly-founded schools in 1985.[8] Most of the corresponding age population came to attend senior high schools, whose education is not compulsory, while the number of students per teacher at the schools became lower and lower. It should be noted that the number of students per teacher did not increase during the period of the so-called second baby boom. This fact may be considered to imply that secondary education in Japan became unconstrained by school facilities and teachers in the 1960s.

In Figure 2, the total fertility rate, crude birth rate, per capita GDP and physical capital stock are depicted.[9] In terms of the "demographic transition" hypothesis, Japan moved into phase III of declining fertility and declining mortality with a rapid decline in the birthrate and a resulting decrease in the population growth rate around 1920. Around 1960, Japan shifted to phase IV of stable low fertility and low mortality, in which the

---

[7] Correspondingly, the Financial Aid Act for Promotion of Private Schools (Shiritsu Gakko Shinkou Josei Ho) was issued in 1975.

[8] In 1965, the ratios of publicly-founded senior high schools and students were above 80%.

[9] Per capita GDP and the capital labor ratio are those estimated in Godo and Hayami (2002: Table 3 on p. 970).

birthrate was stable (see Minami (1986) and Ato (2000)). The total fertility rate declined from about 5 in 1920 to about 2 in 1960, except for the 1940-1945 period of the so-called first baby boom.[10] After 1960 the rate stopped declining so rapidly and hovered around 2, but showed a slight downward trend after 1985. In contrast, the growth rates of per capita GDP as well as per capita stock of capital became much higher after 1960 than before.

The experience of Japan may be summarized briefly as follows. High economic growth followed the expansion of secondary education in several decades, and in the meantime the fertility rate declined rather rapidly. This is consistent with the facts found in Godo and Hayami (2002), who showed that the economic catch-up of Japan, in terms of per capita GDP and the capital/labor ratio, started about sixty years after the educational catch-up in terms of average years of schooling.[11]

3. Model

Along with the economic development process, it is natural to consider the unfolding scenario to be as follows: Initially, there is a constraint on the educational choice of individuals, and then economic development relaxes the constraint and eventually makes it cease to bind. In this section, we first introduce a model without constraints on education.

We consider an overlapping generations economy populated by three-period-lived individuals. Individuals are alike except for their ages. Individual attend schools in the first period, work in the second, and retire in the third. An individual raises children with time inputs and lets them go to school by paying the educational costs in the second, working period. The children have matured within the period and leave their parental

[10] It is sometimes said that the Eugenic Protection Act in 1948 and legalization of artificial abortions in 1949 triggered the rapid decline in the fertility rate. However, the fertility rate seemed to begin to decrease before those policies except for the increase during the War.

[11] Furthermore, Godo (2003) has shown that a similar catch-up pattern can be observed for Korea.

home at the end of the period. The production technology of each firm is homogeneous of degree one in physical capital and effective labor. Physical capital stock depreciates fully after one period of use.

3.1 Individuals

An individual's academic knowledge, h_t, which is obtained through schooling, depends on both the parental education expenditure and the average stock of academic knowledge of the parental generation. Following Lucas (1988) and Glomm and Ravikumar (1992), it is written as

$$h_t = \theta e_{t-1}^{\gamma} \bar{h}_{t-1}^{\delta} \qquad \theta > 0; \gamma, \delta \in (0,1) \tag{1}$$

where h_t denotes the academic knowledge of a worker in period t (who is called generation t), e_{t-1} is the education expenditure per child made by his parent of generation $t-1$, and $\bar{h}_{t-1}$ is the average stock of academic knowledge of generation $t-1$. In the following we assume that $\delta = 1-\gamma$ since our main concern is the balanced growth paths.

The time endowment of an individual in each period is assumed to be one. Individuals spend full time in school in the first period, allocate the time endowment between working and rearing children in the second period, and spend full time in retirement in the third period. The second-period budget equation of an individual of generation t can be written as

$$w_t h_t (1 - \tau n_t) = c_t + e_t n_t + s_t \tag{2}$$

where w_t is the wage rate per unit of labor, measured in academic knowledge; n_t is the number of children he has; τ is rearing time per child, which is assumed to be constant; c_t denotes consumption; e_t is the education expenditure per child; and s_t denotes savings for retirement. Consumption during retirement is then given as

$$d_{t+1} = r_{t+1} s_t \tag{3}$$

where r_{t+1} is the interest rate, plus one, in period $t+1$.

The lifetime utility of the individual is assumed, as is common in the literature on endogenous growth, to be log-linear:

$$U_t = \ln c_t + \varepsilon \ln n_t + \beta \ln e_t + \rho \ln d_{t+1} \tag{4}$$

Following Glomm and Ravikumar (1992), e_t is considered to represent the quality of school which is the bequest to the offspring, while we also assume (unlike them) that individuals derives direct utility from having children. ε and β are positive constants, which are the weights on utilities from having children and from leaving bequests to them, respectively; and ρ denotes the time discount factor ($0 < \rho < 1$). We assume that $\varepsilon - \beta > 0$.[12]

The problem for the individual is to choose consumption during two periods, c_t and d_{t+1}, the number of children, n_t, and education expenditure per child, e_t, so as to maximize the lifetime utility. The first-order conditions for utility maximization are given as

$$c_t: \quad 1/c_t - \mu_t = 0 \tag{5a}$$

$$n_t: \quad \varepsilon / n_t - \mu_t (w_t h_t \tau + e_t) = 0 \tag{5b}$$

$$e_t: \quad \beta / e_t - \mu_t n_t = 0 \tag{5c}$$

$$d_{t+1}: \quad \rho / d_{t+1} - \mu_t / r_{t+1} = 0 \tag{5d}$$

where μ_t is the Lagrange multiplier attached to the lifetime budget constraint, which is obtained by eliminating s_t from (2) and (3). From (5a)-(5d), we have the optimal plans of the individual as

$$s_t = \frac{\rho}{1 + \varepsilon + \rho} w_t h_t \tag{6}$$

$$n_t = \frac{\varepsilon - \beta}{\tau(1 + \varepsilon + \rho)} \tag{7}$$

$$e_t = \frac{\tau\beta}{\varepsilon - \beta} w_t h_t \tag{8}$$

---

12 It is necessary for $n_t > 0$ and $e_t > 0$.

It should be noted that the fertility rate is constant, i.e. $n_t = n$. Therefore, the rate of population growth is also constant, n. We assume that τ is sufficiently small and/or that $\varepsilon - \beta > 0$ is sufficiently great, ensuring $n \geq 1$, and that the sum of the utility weights on his own consumption is not less than the weight on the number of children he has, i.e., $1 + \rho \geq \varepsilon$. The latter assumption implies that an increase in the (potential) income $w_t h_t$ increases the resources allocated to his consumption, $c_t + s_t$, more than the expenditure for children, $(\tau w_t h_t + e_t) n_t$. Savings and the education expenditure per child are linearly increasing in the wage rate per hour, $w_t h_t$.

3.2 Firms

There are a great number of symmetric firms in the economy. The technology of a representative firm i is assumed to be written by the constant-returns-to-scale production function $Y_t^i = F(K_t^i, A_t L_t^i)$, where Y_t^i, K_t^i and L_t^i are output, capital stock and labor employed in period t, respectively, and A_t represents labor productivity. Following Arrow (1962) and Romer (1986), we assume first that a worker of a firm that increases its physical capital simultaneously learns how to produce more efficiently. An increase in a firm's capital stock increases its stock of productive knowledge of workers through learning-by-doing. Second, we assume that each worker's knowledge is a public good that any other worker can access without costs. Therefore, the productivity index of each worker is proportional to the capital/labor ratio of the economy as a whole. As in Grossman and Yanagawa (1993), the index is represented as

$$A_t = \frac{K_t}{L_t} \frac{1}{a} \qquad (a > 0) \tag{9}$$

where K_t and L_t are the aggregate capital stock and labor, respectively, in the economy in period t. We assume here that labor productivity is proportional to the physical capital/labor ratio instead of the per worker stock of physical capital for two reasons: first, balanced growth will be possible as long as there are constant returns to scale in all arguments;[13] and second, it is not implausible that when the academic knowledge of workers and hence the quality of labor is great, labor productivity (of the quality) cannot be raised unless sufficient capital is accumulated correspondingly. As is well known, the interest rate, plus one, and the wage rate per labor in efficient units (denoted by r and ω, respectively) are constant over time:

$$f'(a) = r = r_t \tag{10a}$$

$$f(a) - af'(a) = \omega = \omega_t \tag{10b}$$

where $f(a) = F(a,1)$. Denoting aggregate output in the economy by Y_t, we have $Y_t = [f(a)/a]K_t$, from which we can see from the homogeneity of degree one of the production function that $\partial Y_t / \partial K_t = r + (\omega / a)$. ω / a represents the external return on capital caused by the production knowledge spillovers on labor productivity, which is allocated to labor. Since the stock of human capital of an individual can be considered as the mixture of these two types of knowledge accumulated in different ways, i.e. $A_t h_t$, the wage rate per hour is given as $A_t \omega h_t = w_t h_t$.

3.2 Market equilibrium

Denoting the population of generation t by N_t, the aggregate labor, measured in terms of academic knowledge, can be written as $L_t = (1 - \varpi n)h_t N_t$. Defining $k_t = K_t / N_t$ as per worker stock of physical capital, we have

$$w_t = (\omega / a)(k_t / h_t)/(1 - \varpi n) \tag{11}$$

The equilibrium condition in the capital market in period t is $K_{t+1} = s_t N_t$ or, in per worker terms,

$$nk_{t+1} = s_t \tag{12}$$

From (2), (3), (10), (12), and the linear homogeneity of the production function, we have the resource constraint in period t:

---

[13] Assuming $A_t = (K_t / L_t)^{\xi} / a$ $(1 > \xi > 0)$ instead of (9) does not alter the analysis essentially. However, if $A_t = K_t^{\psi} / a$ $(1 \geq \psi > 0)$, balanced growth paths may not be obtained because of the scale effect. See, for example, Romer (1986).

$$F(K_t, A_t L_t) = c_t N_t + d_t N_{t-1} + e_t n N_t + K_{t+1}$$

or, in per worker terms,

$$[f(a)/a]k_t = c_t + (d_t / n) + e_t n + nk_{t+1}$$

4. Balanced growth paths

We first examine long-term equilibrium without constraints in subsection 4.1, and then one with constraints in the next sub-section.

4.1 Optimal choice of education level without constraints

By the assumption of identical individuals within each generation, the stock of academic knowledge of an individual is equal to the average level of the generation, i.e., $\overline{h}_t = h_t$. Taking (11) into account and substituting e_t from (8) into the academic knowledge production function (1), we obtain

$$h_{t+1} = \theta[\frac{\tau\beta}{\varepsilon - \beta}(\frac{\omega}{a})\frac{1+\varepsilon+\rho}{1+\beta+\rho}]^{\gamma}(\frac{k_t}{h_t})^{\gamma} h_t \quad (13)$$

The growth rate of academic knowledge positively depends on the capital/academic knowledge ratio, k_t / h_t. When physical capital stock per worker is relatively small, and hence the capital/academic knowledge ratio is small, the growth rate of academic knowledge is low, and, conversely, when the capital/academic knowledge ratio is greater, the academic knowledge grows faster.

Taking (11) into account, inserting s_t from (6) into (12) and making use of $n = n_t$ in (7), we have

$$k_{t+1} = [\frac{\tau\rho}{1+\beta+\rho}(\frac{\omega}{a})\frac{1+\varepsilon+\rho}{\varepsilon-\beta}]k_t \quad (14)$$

That is, the growth rate of physical capital stock per worker is constant over time.

Dividing both sides of (14) by those of (13), we obtain

$$(\frac{k_{t+1}}{h_{t+1}}) = \frac{\frac{\tau\rho}{1+\beta+\rho}(\frac{\omega}{a})\frac{1+\varepsilon+\rho}{\varepsilon-\beta}}{\theta[\frac{\tau\beta}{\varepsilon-\beta}(\frac{\omega}{a})\frac{1+\varepsilon+\rho}{1+\beta+\rho}]^{\gamma}}(\frac{k_t}{h_t})^{1-\gamma} \tag{15}$$

Since $0<\gamma<1$, we can see from (15) that the capital/academic knowledge ratio, (k_t/h_t), monotonically converges to a stationary value

$$(\frac{k}{h}) = \left\{ \frac{\frac{\tau\rho}{1+\beta+\rho}(\frac{\omega}{a})\frac{1+\varepsilon+\rho}{\varepsilon-\beta}}{\theta[\frac{\tau\beta}{\varepsilon-\beta}(\frac{\omega}{a})\frac{1+\varepsilon+\rho}{1+\beta+\rho}]^{\gamma}} \right\}^{1/\gamma} \tag{16}$$

Since the growth rate of the capital stock per worker is constant as shown in (14), equation (16) implies that academic knowledge becomes to grow at the same rate as the capital stock per worker in the stationary state.

Defining the balanced growth rate as $g = \frac{Y_{t+1}/N_{t+1}}{Y_t/N_t} - 1$, and since $Y_t = [f(a)/a]K_t$, we obtain

$$1+g = \frac{k_{t+1}}{k_t} = \frac{\tau\rho}{1+\beta+\rho}(\frac{\omega}{a})\frac{1+\varepsilon+\rho}{\varepsilon-\beta} \tag{17}$$

The balanced growth rate is equal to the growth rate of capital stock per worker. Other things being equal, the balanced growth rate is higher when the child-rearing time, τ, is greater, when the difference between the weights to utilities from having children and from bequests left to each child, $\varepsilon-\beta$, is smaller, and/or when the external return on capital caused by the production knowledge spillovers on labor productivity, ω/a, is greater. The intuition behind this result is simple. When child-rearing time is great and/or when the weight to utility from having children is small, the fertility rate will be low, and the number of workers in the next period will be small. When the fraction of the marginal (social) product allocated to workers is great, savings and physical capital accumulation will be great. Since the growth engine in our model is productive knowledge brought about by physical capital accumulation, the acceleration in the accumulation of capital stock per worker raises the growth rate.

On the balanced-growth path, the wage rate per hour, $w_t h_t$, grows at the same rate as the balanced-growth rate, as can be seen from (11). However, since both academic knowledge and physical capital stock are the state variables and the initial values, $(k/h)_0$, are given historically, the growth rate of academic knowledge is not always equal to the growth rate of per capita physical capital, as can be seen from (15). Suppose that the initial capital/academic knowledge ratio, $(k/h)_0$, is smaller than the stationary value, (k/h). Then the ratio monotonically increases as time passes. We can see from (13) that along with this transition, the growth rate of academic knowledge becomes higher, and the wage rate per unit of labor measured in academic knowledge, w_t, will decline as time goes on, although the wage rate per hour, $w_t h_t$, rises at the balanced-growth rate even during the transition.

4.2 Education choice with constraints

In the previous sub-section we have assumed that individuals are not subject to any constraints on educational choices for their children. Considering school education rather than education within the home, the school and/or teacher shortage may set a limit on the level of school education, especially in underdeveloped economies, as stated in the Introduction. For example, as in Japan's experience in Section 2, some schools must be given up because of the lack of financial support. In this section, we assume that the level of school education is constrained in the sense that education services may not necessarily be provided to the extent that each parent considers to be optimal under given income and prices, and that an individual is forced to accept the constrained level which is lower than the optimal level that he would choose without such a constraint. We also assume that the restricted level of education, $\bar{e}_t$, depends on the wage rate per hour:

$$\bar{e}_t = E w_t h_t \qquad (18)$$

where E is a measure of the strength of the constraint and satisfies $0 < E < \tau\beta/(\varepsilon - \beta)$.

Referring to (8), $\bar{e}_t$ is shown to be smaller than the optimal level without the constraint. Although individuals cannot control the constrained level, they must bear the burden of the education cost. The rate of burden is represented by a fraction of the per hour wage rate, E.

Under constraint (18), an individual chooses consumption during working and retirement periods and the number of children so as to maximize lifetime utility subject to the budget constraint. From the first-order conditions for utility maximization, we have

$$c_t = \frac{1}{1+\varepsilon+\rho} w_t h_t \tag{19a}$$

$$w_t h_t \tau n_t + \bar{e}_t n_t = \frac{\varepsilon}{1+\varepsilon+\rho} w_t h_t \tag{19b}$$

$$\frac{d_{t+1}}{r_{t+1}} = \frac{\rho}{1+\varepsilon+\rho} w_t h_t \tag{19c}$$

From (19a), (19b) and the budget equation in the second period, we obtain

$$s_t = \frac{\rho}{1+\varepsilon+\rho} w_t h_t \tag{20}$$

Making use of (18) and (19b), we also have

$$n_t = \frac{\frac{\varepsilon}{1+\varepsilon+\rho} w_t h_t}{w_t h_t \tau + \bar{e}_t} = \frac{\varepsilon}{(\tau+E)(1+\varepsilon+\rho)} [= n_E] \tag{21}$$

Comparing (21) with (7), we can show that $n_E > n$. That is, the fertility rate chosen under the constraint is higher than the one without the constraint.

Assuming that the production sector is the same as the one in the previous section, replacing n in (11) with n_E, taking (21) into account, and rewriting the equilibrium condition in the capital market, $n_E k_{t+1} = s_t$, we obtain

$$k_{t+1} = \frac{\rho(\tau+E)^2(1+\varepsilon+\rho)}{\varepsilon[(\tau+E)(1+\varepsilon+\rho)-\tau\varepsilon]} (\frac{\omega}{a}) k_t \tag{22}$$

Replacing n in (11) with n_E, and from (18), the academic knowledge production function, (1), becomes

$$h_{t+1} = \theta E^{\gamma}[(\frac{\omega}{a})\frac{(\tau+E)(1+\varepsilon+\rho)}{(\tau+E)(1+\varepsilon+\rho)-\tau\varepsilon}]^{\gamma}(\frac{k_t}{h_t})^{\gamma} h_t \tag{23}$$

From (22) and (23), as in the previous sub-section, we have

$$(\frac{k_{t+1}}{h_{t+1}}) = \frac{\frac{\rho(\tau+E)^2(1+\varepsilon+\rho)}{\varepsilon[(\tau+E)(1+\varepsilon+\rho)-\tau\varepsilon]}(\frac{\omega}{a})}{\theta E^{\gamma}[(\frac{\omega}{a})\frac{(\tau+E)(1+\varepsilon+\rho)}{(\tau+E)(1+\varepsilon+\rho)-\tau\varepsilon}]^{\gamma}}(\frac{k_t}{h_t})^{1-\gamma} \tag{24}$$

Therefore, the capital/academic knowledge ratio monotonically converges to the stationary value

$$(\frac{k}{h})_E = \left\{\frac{\frac{\rho(\tau+E)^2(1+\varepsilon+\rho)}{\varepsilon[(\tau+E)(1+\varepsilon+\rho)-\tau\varepsilon]}(\frac{\omega}{a})}{\theta E^{\gamma}[(\frac{\omega}{a})\frac{(\tau+E)(1+\varepsilon+\rho)}{(\tau+E)(1+\varepsilon+\rho)-\tau\varepsilon}]^{\gamma}}\right\}^{1/\gamma} \tag{25}$$

as in the previous sub-section.

The balanced growth rate with the constrained education level, g_E, is obtained in a similar way as in the previous case:

$$1+g_E = \frac{k_{t+1}}{k_t} = \frac{\rho(\tau+E)^2(1+\varepsilon+\rho)}{\varepsilon[(\tau+E)(1+\varepsilon+\rho)-\tau\varepsilon]}(\frac{\omega}{a}) \tag{26}$$

From (17), (26) and the assumption that $1+\rho \geq \varepsilon$, we can easily show that the growth rate under constrained education is lower than the growth rate without constraint, i.e. $g_E < g$. The educational constraint affects fertility through the trade-off between the quality and quantity of children, which in turn changes per capita stock of physical capital and the wage rate per hour. It should be noted that even under the educational constraint, the wage rate per hour, $w_t h_t$, grows at the same rate as the balanced growth rate, g_E.

With the constraint on educational choices of individuals, the growth rate of academic knowledge is not also always equal to the growth rate of per worker capital stock. If the initial capital/academic knowledge ratio, $(k/h)_0$, is higher than the stationary value,

(k/h), the capita/academic knowledge ratio monotonically declines to the stationary value as time passes.

5. Relaxation of educational constraints

We examine the effects of relaxing the educational constraint. Constructing schools and promoting and training teachers require a huge cost, but the average income level of individuals would not be high enough to finance it in the earlier stages of development because of scale indivisibilities or other types of non-convexity of the cost. Even when the educational choices of individuals are constrained, the income level will rise, and people may be able to afford more schools and teachers in the course of development. However, unless the level of education rises more than the income level, individuals would not come to enjoy the optimal level of education. Therefore, governments will be expected to play an important role in relaxing the constraints and expanding the education level. Indeed, for example, Johansson and Mosk (1987) emphasized the government's commitment to public health and the effective measures to lower death rates during the economic development of Japan, while Ministry of Education of Japan (1962) pointed out that the establishment of school system was one of the modernization policies, such as the plan for enriching the nation and building up the defenses, increasing and building up industries and cultural enlightenment, which were undertaken by the Meiji government to catch up with the Western developed countries.[14] On the other hand, the government's policy on school system in the 1960s and 1970s seemed to have followed the education demand of the nation backed by the rapidly growing income.[15] In this section, assuming that the government pushes an expansionary policy on education, we examine the effects of relaxing the

---

[14] In contrast, for example, Galor and Moav (2006) mentioned that in the second phase of the Industrial Revolution in England, capitalists demanded government intervention in the provision of education.

[15] See JICA (2004: Chap. 1) for the evaluation of the education policy in Japan after World War II.

constraints and eventually making them cease to bind, on growth and fertility.[16]

A relaxation of the constraint which makes the rise in the education level higher than the rise in the wage rate can be expressed as an increase in the portion, E. From (26) we have the effect of an increase in E on the balanced growth rate as

$$\frac{d}{dE}(1+g_E) = (\frac{\omega}{a})\frac{\rho}{\varepsilon}\frac{(1+\varepsilon+\rho)(\tau+E)[(\tau+E)(1+\varepsilon+\rho)-2\tau\varepsilon]}{[(\tau+E)(1+\varepsilon+\rho)-\tau\varepsilon]^2}$$

$$> (\frac{\omega}{a})\frac{\rho}{\varepsilon}\frac{2E\varepsilon(1+\varepsilon+\rho)(\tau+E)}{[(\tau+E)(1+\varepsilon+\rho)-\tau\varepsilon]^2}$$

$$> 0 \tag{27}$$

where the second inequality obtains from the assumption that $1+\rho \geq \varepsilon$. The increased fraction spent on education out of the wage rate leads to a rise in the balanced growth rate. The intuition is as follows: The expanded school education enlarges academic knowledge, raises the wage rate per hour, and hence increases savings. An increase in savings accelerates capital accumulation, and thereby hastens economic growth with increasing productive knowledge. It should be noted that the relaxation of the constraint first stimulates academic knowledge accumulation and then leads to productive knowledge accumulation through physical capital accumulation. This process seems consistent with the explanation of Japan's economic development in Godo and Hayami (2002). From (21) we also obtain

$$\frac{dn_E}{dE} = \frac{-\varepsilon}{(1+\varepsilon+\rho)(\tau+E)^2} < 0 \tag{28}$$

An increase in educational cost per child drops the fertility rate and increases the labor supply, $1-\varpi n_E$, since the rearing time per child is constant.

The relaxation of the constraints affects the balanced-growth wage rate in two ways; positively by decreasing the number of workers in the next period and negatively by increasing working time of workers. From (25), we can show that if

[16] We assume implicitly that the costs are financed with school fees and/or taxes, keeping the balanced budget.

$E(1-\gamma)\tau\varepsilon > [(\tau+E)(1+\varepsilon+\rho)-\tau\varepsilon][E(1-\gamma)-\tau\gamma]$ holds, we have $d(k/h)_E/dE<0$. In this case, we can see that $dw/dE<0$ where $w=(\omega/a)(k/h)_E/(1-\tau n_E)$. Both increases in working time and in human capital tends to lower the wage rate per unit of effective labor. On the other hand, when E is small, especially when E is close to zero, we have $E(1-\gamma)\tau\varepsilon < [(\tau+E)(1+\varepsilon+\rho)-\tau\varepsilon][E(1-\gamma)-\tau\gamma]$ and therefore $d(k/h)_E/dE>0$. An increase in E increases the steady state physical capital/human capital ratio. In this case, the wage rate per unit of effective labor can be raised, the increased labor supply of individuals being offset by the increased stock of physical capital.

In the course of economic development, the government may relax constraints in a step-by-step manner in line with rises in the income levels of the economy, perhaps because of the tighter budget. The relaxations in turn continue to raise the balanced growth rate and the income level of the economy, and reduce the fertility rate. However, once E is raised to reach $\tau\beta/(\varepsilon-\beta)$, the constraint on the educational choice of individuals ceases to bind. The decentralized educational choice of individuals does not need government intervention anymore. The growth rate comes to remain at a high level, and the fertility rate will stop decreasing and stay at a constant rate. At that point in time the economy shifts from the constrained paths, analyzed in Section 4.2, to the unconstrained path, analyzed in Section 4.1. It should be noted at this stage that the government in a period relaxes the constraint on the child-rearing generation at that period. Condition (27) means a higher E leads to a higher lifetime utility of far-future generations. However, the government will not increase E beyond $\tau\beta/(\varepsilon-\beta)$ since the child-rearing generation does not want the government to do so, that is, since an increase in E beyond $\tau\beta/(\varepsilon-\beta)$ deteriorates their lifetime welfare.[17]

[17] The life time utility of generation t can be described as $U_t = \ln[w_t h_t/(1+\varepsilon+\rho)] + \varepsilon \ln n_E + \beta \ln(E w_t h_t) + \rho \ln[\rho w_t h_t/(1+\varepsilon+\rho)]$, where $w_t h_t$ is already given at the policy change. Therefore, we have $\partial U_t/\partial E = [\beta\tau - (\varepsilon-\beta)E]/[E(\tau+E)]$ and hence $\partial U_t/\partial E \gtreqless 0$ as $E \lesseqgtr \beta\tau/(\varepsilon-\beta)$.

The time paths of fertility rate, the balanced growth rate and per worker GDP (in logarithmic values) are illustrated in Figure 3. Before time t_0, the constraint on educational choice is binding, the level and the growth rate of income are low, and the fertility rate is high. The constraint is relaxed in a step-by-step fashion until time t_1. During the period of relaxing the constraint, the level and the growth rate of income are rising, while the fertility rate is declining. At time t_1 the constraint ceases to bind. If the relaxation takes in place continuously, the time paths can be approximated by curves.

6. Conclusion

We have first synthesized two types of human capital accumulation, academic education and learning-by-doing with knowledge spillovers, in a single model of endogenous growth, and then, characterizing the constraint on educational choice of individuals due to the school and/or teacher shortage as the feature of a latecomer economy, we have examined the effects of relaxing the constraints on economic development and growth as well as the fertility of the economy. Relaxing constraints enlarges the academic knowledge of individuals and raises the level and the growth rate of income. Through the income effect, savings and hence capital accumulation will be accelerated. Accelerated capital accumulation in turn enlarges productive knowledge through learning-by-doing and knowledge spillovers. Human capital accumulated in these two different ways promotes economic growth. However, expanding education requires more costs, which reduces the fertility rate through the trade-off between the quality and quantity of children. Our prediction from the analysis seems consistent with Japan's experience. Taking the ratio of the number of students to the number of teachers as a proxy variable representing to how much extent the educational constraint is effective, a decline in the number of students per teacher means the elimination of school and/or teacher shortages. In this sense, the constraint seemed to cease to bind around 1965 (see Figure 1). After that time the fertility rate stopped rapidly declining and the growth rate became higher (see Figure 2), although the fertility rate is still decreasing slightly after about 1985.[18]

The important implication of our analysis for latecomer economies, who cannot afford to allocate enough resources to education, is that it is crucial for a society to push expansionary school education policies in earlier stages of development. At the start of the education system in Japan, it was said that the challenge for the government was to narrow the gap between its education policy goals and the people's reluctant participation. But, by around 1920, the new school education system had begun to exert stronger influence after its revisions (Ohkawa and Kohama,1986). Even under this constraint the economy can grow, but it will take a longer time for the economy to be developed.

[18] There are uncertainties and/or various noises on variables in the real world, which seem make the time paths different from the prediction our model makes.

References

Arrow, K. J. (1962) "The Economic Implications of Learning by Doing," *Review of Economic Studies* 29, 155-173.

Ato, M. (2000) *Modern Demography*. Nihon Hyoronsha: Tokyo. (in Japanese)

Barro, R. J. and W.-W. Lee (2000) "International Data on Educational Attainment: Updates and Implications," NBER Working Paper No. 7911, National Bureau of Economic Research.

Becker, G. S. and H. G. Lewis (1973) "On the Interaction between the Quantity and Quality of Children," *Journal of Political Economy* 81, S279-S287.

de Gregorio J. (1996) "Borrowing constraints, human capital accumulation, and growth," Journal of Monetary Economics 37, 49-71.

de la Croix, D. and M. Doepke (2003) "Inequality and Growth: Why Differential Fertility Matters," *American Economic Review* 93, 1091-1113.

Galor, O. (2005) "From Stagnation to Growth: Unified Growth Theory," in: P. Aghion and S. Durlauf (eds.), *Handbook of Economic Growth*, North Holland: Amsterdam, 171-293.

Galor, O. and O. Moav (2004) "From Physical to Human Capital Accumulation: Inequality and the Process of Development," *Review of Economic Studies* 71, 1001-1026.

Galor, O. and O. Moav (2006) "Das Human Kapital: A Theory of the Demise of the Class Structure," *Review of Economic Studies* 73, 85-117.

Galor, O. and D. N. Weil (1999) "From Malthusian Stagnation to Modern Growth," *American Economic Review* 89, 150-154.

Galor, O. and D. N. Weil (2000) "Population, Technology, and Growth: From Malthusian Stagnation to the Demographic Transition and beyond," *American Economic Review* 90, 806-827.

Galor, O. and J. Zeira (1993) "Income distribution and macroeconomics," Review of Economic Studies 60, 35-52.

Glomm, G. and B. Ravikumar (1992) "Public versus Private Investment in Human Capital: Endogenous Growth and Income Inequality," *Journal of Political Economy*

100, 818-834.

Godo, Y. (2003) "Education and economic catch-up," in: K. Otsuka and T. Kurosaki (eds.), *Education and Economic Development*, Toyokeizai Shinposha: Tokyo. (in Japanese)

Godo, Y. and Y. Hayami (2002) "Catching-up in Education in the Economic Catch-up of Japan with the United States, 1890-1990," *Economic Development and Cultural Change* 50, 961-978.

Grossman, G. M. and N. Yanagawa (1993) "Asset Bubbles and Endogenous Growth," *Journal of Monetary Economics* 31, 3-19.

Hanushek, E. A. (1986) "The Economics of Schooling: Production and Efficiency," *Journal of Economic Literature* 24, 1141-1177.

Japan International Cooperation Agency (JICA) (2004) *The History of Japan's Educational Development: What Implications Can Be Drawn for Developing Countries Today*. Institute for International Cooperation, Japan International Cooperation Agency: Tokyo.

Johansson, S. R. and C. Mosk (1987) "Exposure, Resistance and Life Expectancy: Disease and Death During the Economic Development of Japan, 1900-1960," *Population Studies* 41, 207-235.

Kaser, M. C. (1966) "Education and Economic Progress: Experience in Industrialized Market Economies," in R. Edward Austin Gossage and J.E. Vaizey (eds.), *The Economics of Education*, MacMillan Publishers: London.

Kruger, A. B. and M. Lindahl (2001) "Education for Growth: Why and for Whom?," *Journal of Economic Literature* 39, 1101-1136.

Lee, J.-W. and R. J. Barro (2001) "Schooling Quality in a Cross-section of Countries," *Economica* 68, 465-488.

Lucas, R. E. (1988) "On the Mechanics of Economic Development," *Journal of Monetary Economics* 22, 3-42.

Maddison, A. (2001) The World Economy – A Millennial Perspective. *Development Centre Studies*, OECD: Paris.

Matthews, R. C. O., C. H. Feinstein and J. C. Odling-Smee (1982) *British Economic Growth 1856-1973*. Stanford University Press: Stanford.

Minami, R. (1986) *The Economic Development of Japan*. MacMillan Publishers: London.

Ministry of Education of Japan (1954) *Eighty-Year History of Japanese Education System.* (in Japanese)

Ministry of Education of Japan (1962) *Japan's Growth and Education.* (in Japanese)

Moav, O. (2005) "Cheap Children and the Persistence of Poverty," *Economic Journal* 115, 88-110.

Ohkawa, K. and H. Kohama (1986) *Lectures on Developing Economies: Japan's Experience and Its Relevance*. University of Tokyo Press: Tokyo.

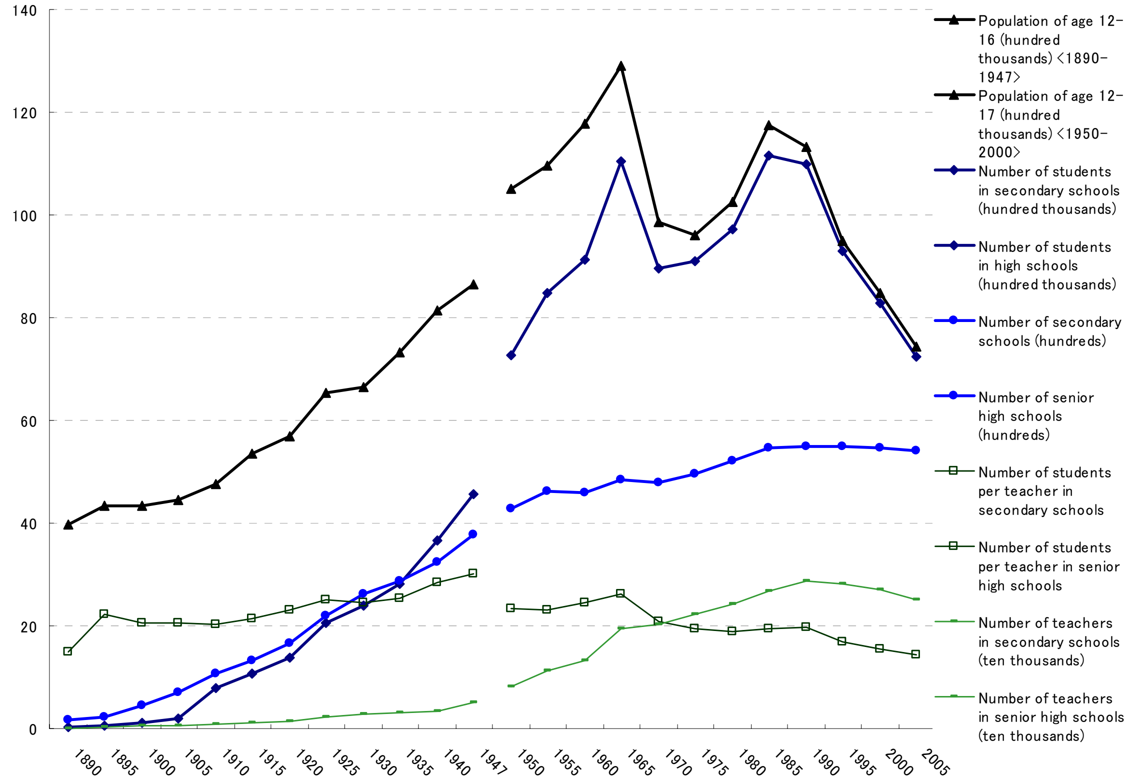
Romer, P. M. (1986) "Increasing Returns and Long-run Growth," *Journal of Political Economy* 94, 1002-1037.

Sato, M. (2004) "Financial Commitment of Government on Educational Development in Developing Countries," in K. Ohtsu (ed.) *Studies on Fundamental Education Curriculum and Program in Developing Countries*. Final Report of the Grant-in-Aid of Ministry of Education, Culture, Sports, Science and Technology of Japan: Hokkaido University of Education Sapporo, pp. 273-282. (in Japanese)

Terao, T. (1956) *Economics of Education*. Dobunkan: Tokyo. (in Japanese)

World Bank (1995) *Priorities and Strategies for Education: A World Bank Review*. The World Bank: Washington D.C.

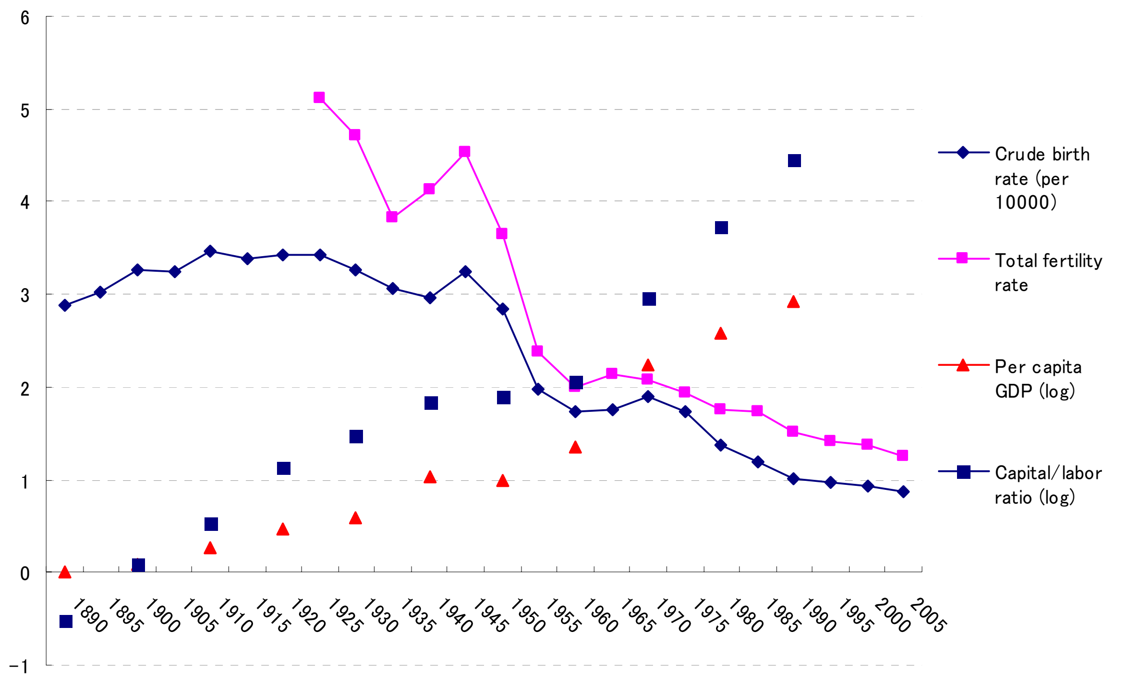
FIGURE 1. School age population, numbers of students, teachers and schools, and number of students per teacher



Sources: School age population : Ministry of Internal Affairs and Communications (2005). Long-term Statistics Series of Japan (Chap. 2 Population and Households).
Others : Ministry of Education of Japan (1962); School Basic Survey: Annual Statistics (Ministry of Education, Culture, Sports, Science and Technology of Japan, 2008)

Notes: 1. Populations at every 5 years from 1890 to 1915 are those from 1888 to 1913.
2. Secondary schools contain Kyusei-Chugaku, Kouto Jogakko, Jitugyo Gakko and Shihan Gakko.

FIGURE 2. Total fertility rate, birth rate, per capita GDP and capital/labor ratio



Sources: Crude birth rate : Ministry of Internal Affairs and Communications (2006). Long-term Statistics Series of Japan (Chap. 2 Population and Households).
Total fertility rate : National Institute of Population and Social Security (2009). Population Statistics of Japan.
Per capita GDP, capital/labor ratio : Godo and Hayami (2002)

Notes: 1. Birthrates are moving 7-year averages with adjustment for 1944-6 and 2000 because of the absence of data.
2. Total fertility rate before 1970 does not include Okinawa.
3. GDP and capital stock are evaluated at 1990 price and measured in 1000 US dollars.

FIGURE 3. Time paths －fertility rate, balanced growth rate, per worker GDP－