DISCUSSION PAPERS ON DEMOGRAPHY AND POLICY

#2010-001

Fertility, intra-generational income redistribution and social security reform

Makoto Hirazawa
Graduate School of Economics, Chukyo University

Koji Kitaura
Department of Civil Engineering, Gifu University

Akira Yakita
Graduate School of Economics, Nagoya City University

July 2010

HITOTSUBASHI UNIVERSITY

Public Economics Program
School of International and Public Policy
Tokyo, Japan

Fertility, intra-generational income redistribution and social security reform

Makoto Hirazawa[a], Koji Kitaura[b], Akira Yakita[c*]

[a] *Graduate School of Economics, Chukyo University, Japan*

[b] *Department of Civil Engineering, Gifu University, Japan*

[c] *Graduate School of Economics, Nagoya City University, Japan*

ABSTRACT

Incorporating heterogeneity in preference for having children into a small open overlapping generations model, we examine the effects of changes in the size of PAYG social security on fertility choices of individuals and population growth of the economy under different degrees of intra-generational redistribution. It is shown that PAYG social security will raise population growth by increasing the number of individuals who have children if the system involves redistribution between retirees with different contributions, while, if it has no redistribution, PAYG social security can decrease the number of children, reducing the number of future contributors to the system.

Keywords: Pay-as-you-go social security; Fertility; Intra-generational redistribution; Dynamic resource allocation

JEL classification: H55; J13; J14; J18

* Corresponding author: Akira Yakita, Graduate School of Economics, Nagoya City University, 1 Yamanohata, Mizuho-cho, Mizuho-ku, Nagoya 467-8501, Japan.
E-mail address: yakita@econ.nagoya-cu.ac.jp (A. Yakita).

1. Introduction

Declining fertility rates put pressures on the financing of pay-as-you-go (PAYG) social security systems, especially, in Western countries. The fertility decline may also endanger the sustainability of the society itself because of the shrinking population (e.g., Cigno, 1993; Sinn, 2004). The cost of rearing children must be shouldered by each household, while the size of a person's pension benefits depend on everyone else's fertility decisions, giving some individuals the incentive for free-riding by receiving benefits without paying the cost (e.g., Folbre, 1994). Such free-riding is considered to exert negative effects on the fertility decisions of individuals. Thus, social security reforms have been proposed to resolve the problem by, for example, conversion of social security benefits to a parental dividend (Bental, 1989; Burggraf, 1993), to a (voluntary) self-financing social security program that promises a return equal to the individual fertility rate (Eckstein and Wolpin, 1985) or to a PAYG social security cum child allowance system (Groezen et al., 2003). As a matter of fact, the social security systems (public pensions) employed in most developed countries involve some degree of such intra-generational redistribution, although the works cited above assumed homogeneous individuals. For example, a flat rate benefit system partly financed by consumption taxes will be introduced in Japan (e.g., the 2004 Revision).[1] The flat rate benefit scheme involves intra-generational redistribution when individuals are heterogeneous, while consumption taxes will stimulate private savings for retirement and hence capital formation.

Our purpose in the present study is to examine the effects of PAYG social security on the sustainability of the social security system as well as the society itself through changes in fertility decisions of individuals, that is, decisions of whether or not to have children, while focusing on the intra-generational redistribution through pension

[1] It is often said that the trend of social security reform in the world is the switch from defined-benefit to defined-contribution systems. In contrast, the reform in Japan can be said to maintain the property of "collective annuities" à la Cremer et al. (2010) through a defined-benefit system.

benefits between individuals with and without children. The longer the child-rearing time, the shorter the working time, and hence the lower the contribution to PAYG social security. Most of the literature has not explicitly taken into account the effects of the intra-generational redistribution through PAYG social security benefit on fertility decisions of individuals in considering reforms of PAYG social security. There are many works assuming heterogeneous agents.[2] The paper closest to our own is that of Cremer et al. (2008), who showed that, assuming both endogenous fertility and heterogeneity in the ability to raise children, the optimal PAYG schemes require a marginal subsidy on fertility to correct for the externality under perfect information and additional subsidy depending on whether the redistribution is geared more to people with more children. However, they did not consider the forgone income of child-rearing. The present paper takes into account the trade-off in time allocation between market work and child rearing, which is important in the present context if the child-rearing cost consists of units of parents' time, i.e., forgone income of parents, and units of goods, i.e., investment in the "quality" of children (Barro and Becker 1989, p. 486).

We show that if the benefit level is not linked to the contribution, i.e., under the Beveridgean benefit scheme, a rise in the contribution rate increases the fertility rate. In this case, therefore, it enhances the sustainability of the social security system and the society in the sense that the supporters of both in the future will increase. On the other hand, if the benefit level is proportional to the contribution, i.e., under the Bismarckian benefit scheme, whether or not a rise in the contribution rate increases the

[2] For example, assuming both wage inequality and longevity difference among individuals, Cremer et al. (2010) showed that when the former dominates, a flat rate benefit (Beveridgean) system is more welfare improving than a contribution (Bismarckian) system. Cremer et al. (2004) also showed that, assuming heterogeneous individuals in the levels of productivity and health status, redistribution through social security may impose an implicit tax on postponed retirement, thus inducing early retirement for some individuals. In the present study, we assume inequality in contributions due to differences in the lengths of working time rather than wage inequality.

fertility rate depends on the relative magnitudes of the interest rate, i.e., the rate of return to private savings, and the population growth rate, i.e., the rate of return to PAYG social security contributions. When the population growth rate is higher than the interest rate, i.e., when the resource allocation is dynamically inefficient, individuals seek to increase future income by increasing social security contributions through supplying more labor to the market and reducing child-rearing time, therefore reducing the fertility rate. In the reverse case, individuals switch their time from market labor to child-rearing, thereby increasing the fertility rate.

This paper is organized as follows. The next section introduces the model and Section 3 examines the effect of a PAYG social security system on the population growth rate of the society. We assume a defined-contribution system in the present study. Section 4 examines the welfare effects of a rise in the contribution rate of PAYG social security. A final section concludes the paper.

2. Model

We consider a small open economy facing the world interest rate, r, which is assumed to remain constant over time. Assuming a standard neoclassical constant-returns-to-scale production function and perfectly competitive factor markets, the wage rate, w, is also constant.

The economy is populated by overlapping generations of people who live for three periods. Each individual is reared by his parent in the first period of life, works and possibly rears children in the second, and retires in the third. Individuals in each generation differ only in their preference for having children. The degree of the preference of an individual is represented by the (marginal) utility weight, α, of having children relative to material consumption.[3] We assume that α is distributed

[3] Alternatively, we may assume that α denotes the probability of having children.

over $[0, \overline{\alpha}]$ according to the cumulative distribution function $F(\alpha) = \int_0^{\alpha} f(x)dx$ where $f(x)$ is the density function and $F(\overline{\alpha}) = 1$. The distribution is assumed to be the same for every generation, though the population size may change over time. For expositional purpose, we assume that each individual has only two options, i.e., having a given number of children $\tilde{n}(\geq 1)$ or having none at all.[4]

Normalizing the time endowment during the working period to one, and assuming that the rearing time per child ε is constant, the budget constraints of individuals with and without children in the second period are given, respectively, as

$$s_t^P = (1-\tau)w(1-\varepsilon\tilde{n}) \tag{1a}$$

$$s_t^N = (1-\tau)w \tag{1b}$$

where s_t^j denotes the savings of a working individual ($j = P, N$) and τ is the social security contribution rate in each period.[5] The superscripts, P and N, denote individuals with and without children, respectively. The budget constraints in the third period can be given, respectively, as

$$c_{t+1}^P = R(1-\tau)w(1-\varepsilon\tilde{n}) + \beta_{t+1}^P \tag{2a}$$

$$c_{t+1}^N = R(1-\tau)w + \beta_{t+1}^N \tag{2b}$$

where $R = 1 + r$ is the gross rate of interest and β_{t+1}^j $(j = P, N)$ denotes the social security benefits in period $t+1$. Letting $\beta_{t+1}^N = \beta_{t+1}^P / \sigma$, parameter σ reflects the extent of intra-generational redistribution between retirees with working (contribution)

---

4 In Japan, for example, the most frequent number of children per household is 2 for all income levels for households with wives aged 40 to 49. For the income class of less than 4 million yen per year, the highest ratio of households with 2 children is 41.4% and then that of 1 child is 19.0%; for the income class between 4 and 6 million yen per year, the highest ratio of 2 children is 48.1% and then that of 1 child is 20.2%; for the income class between 6 and 8 million yen per year, the highest ratio of 2 children is 50.7% and then that of a child is 20.9%; for the income class between 8 and 10 million yen per year, the highest ratio of 2 children is 48% and then that of a child is 22.5%; and for the income class of more than 10 million yen per year, the highest ratio with 2 children is 43.2% and then that of a child is 23.7%. (Cabinet Office, Government of Japan, 2005: http://www5.cao.go.jp/seikatsu/whitepaper/ h17/01_honpen/image/hm020109.gif, cited on 24 June 2010)

5 We assume that $1-\varepsilon\tilde{n} > 0$. Since, if $\varepsilon = 0.075$ as in de la Croix and Doepke (2003), we have $1/\varepsilon = 13.33$, the condition holds plausibly.

periods varying $1-\varepsilon\tilde{n}$ to 1. We restrict our attention to the range $1-\varepsilon\tilde{n} \leq \sigma \leq 1$ for our analytical purpose. The case of $\sigma = 1$ corresponds to perfect redistribution within a generation (i.e., a flat rate benefit scheme), while the case of $\sigma = 1-\varepsilon\tilde{n}$ involves no redistribution (i.e., a contribution-proportional benefit scheme).

For analytical convenience, we assume the utility function of an individual with α to be additively separable and the felicity functions to be linear:

$$U_t = \alpha\tilde{n} + \rho c_{t+1} \tag{3}$$

where ρ is the subjective discount factor. The problem for the individual is to decide to have children or not. We can show that an individual with $\alpha = 0$ does not choose to have children: even when $\sigma = 1$, i.e., even when $\beta_{t+1}^P = \beta_{t+1}^N$, we have

$$\rho[R(1-\tau)w(1-\varepsilon\tilde{n}) + \beta_{t+1}^P] < \rho[R(1-\tau) + \beta_{t+1}^N] \tag{4}$$

where the left-hand side is the lifetime utility when having children and the right-hand side is the utility without children. Therefore, individuals with $\alpha = 0$ do not have children for $\sigma \in [1-\varepsilon\tilde{n}, 1]$.[6] On the other hand, we assume that the individual with the strongest preference for having children $\overline{\alpha}$ chooses to have children, i.e.,

$$\overline{\alpha}\tilde{n} + \rho[R(1-\tau)w(1-\varepsilon\tilde{n}) + \sigma\beta_{t+1}] > \rho[R(1-\tau) + \beta_{t+1}] \tag{5}$$

where $\beta_{t+1} = \beta_{t+1}^N$ (we use this notation for the benefit in the following if it is not confusing). Assuming here that $\overline{\alpha}$ is sufficiently large to satisfy (5), there is a cutoff degree of the preference $\alpha = \alpha^*$, which satisfies

$$\alpha^*\tilde{n} + \rho[R(1-\tau)w(1-\varepsilon\tilde{n}) + \sigma\beta_{t+1}] = \rho[R(1-\tau)w + \beta_{t+1}]$$

or

$$\alpha^* = \frac{\rho}{\tilde{n}}[R(1-\tau)w\varepsilon\tilde{n} + (1-\sigma)\beta_{t+1}]. \tag{6}$$

Only individuals with $\alpha \in [\alpha^*, \overline{\alpha}]$ will have children, and those with $\alpha \in [0, \alpha^*)$ will receive the social security benefits without paying the costs of child-rearing.

---

[6] However, if $\sigma > 1$ is otherwise allowed, $\beta_{t+1}^N < \beta_{t+1}^P$ can hold. In this case condition (4) can be violated, and all individuals may choose to have children and contribute the same amount to the social security system.

Since individuals with the degree of preference $\alpha \geq \alpha^*$ have $\tilde{n}$ children, the evolution of the total population of this economy is

$$N_{t+1} = [\int_{\alpha^*}^{\overline{\alpha}} \tilde{n} dF(\alpha)] N_t \equiv \nu_t N_t \tag{7}$$

where $\nu_t = N_{t+1} / N_t$ denotes the rate of population growth and N_t is the population of the working generation in period t.

The public authority, operating an unfunded social security system, determines the benefit levels so as to balance the budget equation in each period:

$$\tau w L_t = \beta_t [\int_0^{\alpha^*} dF(\alpha) + \sigma \int_{\alpha^*}^{\overline{\alpha}} dF(\alpha)] N_{t-1} \tag{8}$$

where L_t stands for the labor supply in the economy as a whole in period t:

$$L_t = [\int_0^{\alpha^*} dF(\alpha) + (1 - \varepsilon \tilde{n}) \int_{\alpha^*}^{\overline{\alpha}} dF(\alpha)] N_t \ ,$$

that is, the sum of the labor supply of individuals with and without children. From (8), the social security benefit level per retiree can be written as[7]

$$\beta_t = \tau w \frac{\int_0^{\alpha^*} dF(\alpha) + (1 - \varepsilon \tilde{n}) \int_{\alpha^*}^{\overline{\alpha}} dF(\alpha)}{\int_0^{\alpha^*} dF(\alpha) + \sigma \int_{\alpha^*}^{\overline{\alpha}} dF(\alpha)} \nu_{t-1}. \tag{9}$$

3. Changes in contribution rate

Now we examine the effect of a change in the social security contribution rate on the cut-off degree, α^*. Differentiating (6) with respect to τ, we obtain

$$\frac{d\alpha^*}{d\tau} = \frac{\rho}{\tilde{n}} [-Rw\varepsilon\tilde{n} + (1 - \sigma) \frac{d\beta_{t+1}}{d\tau}] \tag{10}$$

[7] Precisely speaking, we are here concerned with the long-term equilibrium, i.e., a constant α^* supported by (τ, β_{t+1}) satisfying (9) for period $t = 0,1,2,\ldots$. While we can show the existence of such a long-term equilibrium, we assume its uniqueness and stability. The proof of the existence is available upon request from the authors.

where, from (7) and (9), we have

$$\frac{d\beta_{t+1}}{d\tau} = \frac{\beta_{t+1}}{\tau} - \beta_{t+1} \frac{(1-\sigma) f(\alpha^*)}{\int_0^{\alpha^*} dF(\alpha) + \sigma \int_{\alpha^*}^{\overline{\alpha}} dF(\alpha)} \frac{d\alpha^*}{d\tau}$$
$$- \frac{\tau w \tilde{n} f(\alpha^*)}{\int_0^{\alpha^*} dF(\alpha) + \sigma \int_{\alpha^*}^{\overline{\alpha}} dF(\alpha)} \{[\int_0^{\alpha^*} dF(\alpha) + (1-\varepsilon\tilde{n}) \int_{\alpha^*}^{\overline{\alpha}} dF(\alpha)] - \varepsilon v_t\} \frac{d\alpha^*}{d\tau}. \tag{11}$$

Substituting (11) into (10) and rearranging terms, we obtain

$$\frac{d\alpha^*}{d\tau} = D^{-1} \frac{R}{\tau} [(1-\sigma) \frac{\beta_{t+1}}{R} - \tau w \varepsilon \tilde{n}] \tag{12}$$

where

$$D \equiv \frac{\tilde{n}}{\rho} + (1-\sigma)\beta_{t+1} \frac{(1-\sigma) f(\alpha^*)}{\int_0^{\alpha^*} dF(\alpha) + \sigma \int_{\alpha^*}^{\overline{\alpha}} dF(\alpha)}$$
$$+ (1-\sigma) \frac{\tau w \tilde{n} f(\alpha^*)}{\int_0^{\alpha^*} dF(\alpha) + \sigma \int_{\alpha^*}^{\overline{\alpha}} dF(\alpha)} \{[\int_0^{\alpha^*} dF(\alpha) + (1-\varepsilon\tilde{n}) \int_{\alpha^*}^{\overline{\alpha}} dF(\alpha)] - \varepsilon v_t\}.$$

Now it should be noted that $D = \tilde{n} / \rho > 0$ when $\sigma = 1$. So we assume $D > 0$ in order to guarantee that $d\alpha^* / d\tau$ is continuous in τ for $\sigma \in [1-\varepsilon\tilde{n}, 1]$. Then, we have the following relations:

$$\frac{d\alpha^*}{d\tau} \gtreqless 0 \quad \text{as} \quad (1-\sigma) \frac{\beta_{t+1}}{R} \gtreqless \tau w \varepsilon \tilde{n} \tag{13}$$

where $(1-\sigma)\beta_{t+1} / R = \beta_{t+1}^N / R - \beta_{t+1}^P / R$ is the relative advantage of having no children in terms of the present values of benefits and $\tau w \varepsilon \tilde{n} = \tau w - (1-\varepsilon\tilde{n})\tau w$ is the relative burden (cost) of having no children in terms of contributions, respectively, at the contribution rate τ. Therefore, if the relative benefit of having no children is greater than the burden under the PAYG social security system, the individual(s) with the cut-off degree may choose to have no children in order to have a greater advantage when the contribution rate rises, that is, the cut-off degree becomes higher. In contrast, when the relative benefit is smaller, the cut-off degree becomes lower and hence individuals with lower preference for children will choose to have children in order to avoid the greater burden as the contribution rate rises.

As can be seen from (13), the relative benefit of having no children depends on the extent of intra-generational redistribution, σ. The heavier the redistribution (i.e., the greater σ), the smaller the relative benefit of having no children; and vice versa. For expositional purpose, we focus on the following two cases: (i) $\sigma = 1$ and (ii) $\sigma = 1 - \varepsilon\tilde{n}$. The former case corresponds to a flat rate (Beveridgean) benefit scheme and the latter is the contribution-proportional rate (Bismarckian) benefit scheme.[8]

In case (i), the condition in (13) becomes $0 < \tau w \varepsilon \tilde{n}$ and, therefore, we have

$$\frac{d\alpha^*}{d\tau} < 0.$$

Although the increased contribution does not change the relative benefit of having children, it only raises the relative cost of having no children. Therefore, individuals with child preference near but lower than the cut-off degree will choose to have children. As a consequence, the fertility rate of the economy rises.

Next, in case (ii), using (9), we can rewrite (13) as

$$\frac{d\alpha^*}{d\tau} \overset{>}{\underset{<}{=}} 0 \quad \text{as} \quad \nu_t \overset{>}{\underset{<}{=}} R. \tag{14}$$

In this case, the population growth rate is the rate of return to compulsory savings of social security. If the population growth rate $(\nu_t - 1)$ is equal to the interest rate $(r = R - 1)$, an increase in the contribution rate does not affect the fertility decisions of individuals since the increased compulsory savings are completely offset by decreases in private savings. When the population growth rate is higher than the interest rate, the individual with the cut-off degree chooses to work more and contribute more since the rate of return to social security is higher. Therefore, since fewer individuals have children, the fertility rate declines (i.e., $d\nu_t / d\tau = -\tilde{n} f(\alpha^*)(d\alpha^* / d\tau) < 0$). However, it should be noted that the lower population growth improves the resource allocation in

[8] The case of homogeneous individuals can be considered to correspond to case (ii) since the benefit/cost ratios are the same among individuals with and without children.

this situation of dynamic inefficiency. In contrast, when the population growth rate is lower than the interest rate, the individual with child preference near but lower than the cut-off degree will save the social security contribution by decreasing the market labor supply and hence choosing to have children, in response to increases in the contribution rate. In this case, the fertility rate rises since more individuals choose to have children. This in turn raises the rate of population growth, improving the dynamic resource allocation. Thus, under the Bismarckian benefit scheme, the PAYG social security system can improve dynamic resource allocation through changing the fertility rate in the economy. This result has not been noticed in the literature.

We can summarize the above arguments as follows:[9]

Result 1

(1) If the social security system involves perfect redistribution between beneficiaries with different contributions, an expansion of social security reduces the cut-off degree, thereby raising the population growth rate of the economy through increasing the number of parents who have children.

(2) If the social security system involves no redistribution among retirees, then the effect of an expansion of the system on the population growth rate of the economy depends on the relative magnitudes of the rate of interest and the population growth rate. When the population is growing faster (slower), a rise in the contribution rate causes a decrease (an increase) in the population growth rate. The increased contribution rate improves the dynamic resource allocation through changing the number of individuals who have children.

---

[9] Our results crucially depend on the assumption of the payroll tax for social security. However, it is not implausible, and is even common in the literature, to assume non-lump-sum contributions. See, for example, Sinn (2004) and Zhang et al. (2001), although Bental (1989) and Groezen et al. (2003) assumed lump-sum taxes. Samuelson (1975a) showed the effect of PAYG social security on the dynamic resource allocation assuming a lump-sum contribution with identical individuals. In the real world, for instance, Sweden introduced a proportional tax in 1999, while Japan has adopted different schemes with lump-sum and proportional contributions.

It should be noted that, in contrast to the previous studies (e.g., Zhang et al., 2001; Yakita, 2001), an increase in the after-tax wage rate may reduce the fertility rate of the society, even though children are a superior "consumption" in our model, and that it is the case even under a Bismarckian benefit scheme.[10]

4. Discussion: welfare effects

In this section, we briefly examine the welfare effects of a change in the social security contribution rate. Inserting $d\alpha^*/d\tau$ from (10) into (11), we have

$$\frac{d\beta_{t+1}}{d\tau} = \frac{\beta_{t+1}}{\tau} - A\frac{\rho}{\tilde{n}}[(1-\sigma)\frac{d\beta_{t+1}}{d\tau} - Rw\varepsilon\tilde{n}] \tag{15}$$

where

$$A \equiv \tau w\tilde{n} f(\alpha^*)\frac{[\int_0^{\alpha^*} dF(\alpha) + (1-\varepsilon\tilde{n})\int_{\alpha^*}^{\overline{\alpha}} dF(\alpha)] - \varepsilon v_t}{\int_0^{\alpha^*} dF(\alpha) + \sigma\int_{\alpha^*}^{\overline{\alpha}} dF(\alpha)} + \frac{\beta_{t+1}(1-\sigma) f(\alpha^*)}{\int_0^{\alpha^*} dF(\alpha) + \sigma\int_{\alpha^*}^{\overline{\alpha}} dF(\alpha)}. \tag{16}$$

The numerator of the first term of A can be rewritten as

$$[\int_0^{\alpha^*} dF(\alpha) + (1-\varepsilon\tilde{n})\int_{\alpha^*}^{\overline{\alpha}} dF(\alpha)] - \varepsilon v_t = \frac{[\int_0^{\alpha^*} dF(\alpha) + (1-\varepsilon\tilde{n})\int_{\alpha^*}^{\overline{\alpha}} dF(\alpha)]N_t - \varepsilon v_t N_t}{N_t} \tag{17}$$

where the first term on the numerator on the right-hand side of (17) denotes the labor supply and the second term, $\varepsilon v_t N_t = \varepsilon\int_{\alpha^*}^{\overline{\alpha}} dF(\alpha)\cdot N_t$, is the child rearing time,

[10] Galor and Weil (1996) emphasized the effect of an increase in women's relative wages in lowering fertility, taking into account the differences between men and women.

respectively, in the economy as a whole. Since it seems plausible that the aggregate labor supply is greater than the aggregate child-rearing time in the economy as a whole, we assume that $A > 0$. Therefore, from (15), we have

$$\frac{d\beta_{t+1}}{d\tau} = (\frac{\beta_{t+1}}{\tau} + A\rho\varepsilon wR)/[1 + A\frac{\rho(1-\sigma)}{\tilde{n}}] > 0. \tag{18}$$

That is, an increase in the contribution rate always raises the benefit level.

The welfare changes for individuals with and without children are given, respectively, by

$$\frac{1}{\rho}\frac{dU^N}{d\tau} = -Rw + \frac{d\beta_{t+1}}{d\tau} \tag{19a}$$

$$\frac{1}{\rho}\frac{dU^P}{d\tau} = -Rw(1-\varepsilon\tilde{n}) + \sigma\frac{d\beta_{t+1}}{d\tau}. \tag{19b}$$

As in the previous section, we consider the following two cases:

(i) case $\sigma = 1$ (flat rate benefit scheme)

In this case, the welfare effects of an increase in the contribution rate are ambiguous. For individuals without children, (19a) becomes

$$\frac{1}{\rho}\frac{dU^N}{d\tau} = Rw(A\rho\varepsilon - 1) + \frac{\beta_{t+1}}{\tau}. \tag{20a}$$

The sign of the first term on the right-hand side of (20a) is ambiguous, and that of the second term is positive. Therefore, the welfare effect on the individuals is ambiguous. However, from (19a), we can show that it is positive (negative) when the benefits are significantly (insignificantly, respectively) raised by an increase in the contribution rate. On the other hand, the welfare effect on the individuals with children is given as

$$\frac{1}{\rho}\frac{dU^P}{d\tau} = Rw[(A\rho\varepsilon - 1) + \varepsilon\tilde{n}] + \frac{\beta_{t+1}}{\tau}. \tag{20b}$$

Therefore, we can not also determine the sign of (20b) a priori. From (20a) and (20b), we have

$$\frac{dU^P}{d\tau} - \frac{dU^N}{d\tau} = \rho w R \varepsilon \tilde{n} > 0. \tag{21}$$

That is, even when $dU^N / d\tau < 0$, we may have $dU^P / d\tau > 0$. As shown in the previous section, when $\sigma = 1$, an increase in the contribution rate raises the fertility rate. Inequality in (21) may reflect the result.

(ii) case $\sigma = 1 - \varepsilon \tilde{n}$ (contribution-proportional benefit scheme)

In this case, we have $\mathrm{sgn}[dU^N / d\tau] = \mathrm{sgn}[dU^P / d\tau] = \mathrm{sgn}[-Rw + d\beta_{t+1} / d\tau]$ from (19a) and (19b). From the results in the previous section, we have $d\alpha^* / d\tau > 0$ ($d\alpha^* / d\tau < 0$) and, from (10), $-wR + d\beta_{t+1} / d\tau > 0$ ($-wR + d\beta_{t+1} / d\tau < 0$) when $\nu_t > R$ ($\nu_t < R$, respectively). Therefore, we have $dU^N / d\tau > 0$ and $dU^P / d\tau > 0$ ($dU^N / d\tau < 0$ and $dU^P / d\tau < 0$) when $d\alpha^* / d\tau > 0$ ($d\alpha^* / d\tau < 0$, respectively). Thus, when an increase in the contribution rate raises (lowers) the fertility rate, the increased contribution rate decreases (increases, respectively) the utility of both individuals with and without children. These results can be interpreted as follows. When resource allocation is dynamically inefficient in the sense that the interest rate is lower than the growth rate of population, an increase in the contribution of PAYG social security increases the welfare of all individuals by enlarging the marginal increase in the social security benefit (see (10)), while it improves the dynamic resource allocation through lowering the fertility rate. On the other hand, when the dynamic resource allocation is efficient, an increase in the contribution rate lowers the welfare of both individuals with and without children through lessening the marginal increase in the benefit, although it raises the fertility rate. It should be noted that in both cases the dynamic resource allocation moves toward to the golden rule of accumulation, changing the population growth rate. In this sense, there can be a negative relationship between the efficiency of resource allocation and fertility (or the sustainability of the society in the sense of the maintenance of population).[11]

---

[11] Assuming uni-sex individuals without infant mortality, the present study supposes that the sustainable growth rate of population is 1. Since not all the individuals have

It should be noted that, in the present setting of heterogeneous individuals and endogenous fertility, downsizing the PAYG social security system does not also yield a Pareto-improvement (see (18)), although the long-term utility of individuals will be higher. This result can be considered to extend the result in Groezen et al. (2003) to a more general case of heterogeneous individuals, although we assume non-lump-sum contributions.[12] Therefore, the notion that a lower PAYG tax would be optimal in the long term when the economy is characterized by dynamic efficiency will not be justified even if the benefit scheme is the contribution-proportional one. Downsizing the PAYG social security may raise fertility in the economy only when the economy is characterized by dynamic inefficiency and under a contribution-proportional benefit scheme.

5. Concluding remarks

Assuming heterogeneity of individuals with respect to the preference for having children in a small open, overlapping generations model, we examined the effect of a change in the size of the PAYG social security system on fertility and the welfare through redistribution among beneficiaries with different contributions (due to child rearing). Although redistribution from individuals without children and with high contributions to those with children and with low contributions raises the fertility rate in the economy, the redistribution may not necessarily lower the welfare of heavy contributors. If social security benefits do not involve redistribution among retirees, there may be a trade-off between the efficiency of dynamic resource allocation and fertility.[13]

---

children, the population growth rate (ν_t) can be less than 1, although $\tilde{n} \geq 1$. If the long-term growth rate of population is less than zero, the economy eventually disappears.

[12] For the case of exogenous fertility, see, for example, Breyer (1989).

[13] Samuelson (1975b) showed the trade-off, assuming identical individuals.

We have not analyzed the transition path after the social security reform. In that sense our analysis remains essentially comparative statics. During the transition from a PAYG social security system with a redistribution scheme to another such system with a different redistribution scheme, the contribution-benefit combinations of the system can be adjusted so as to balance the budget of the authority or in such a manner that the budget balance may be loosened temporarily. These transitional adjustments will affect fertility decisions of individuals and, therefore, the transition path of the economy.[14] Thus, taking this transition into account will be an important issue in the future.

---

[14] For endogenous population growth and fertility decisions, see, for example, Barro and Becker (1989) and de la Croix and Doepke (2003).

References

Barro, Robert J., Becker, Gary S., 1989. Fertility choice in a model of economic growth. Econometrica 57, 481-501.

Bental, Benjamin, 1989. The old age security hypothesis and optimal population growth. Journal of Population Economics 1, 285-301.

Breyer, Friedrich, 1989. On the intergenerational Pareto efficiency of pay-as-you-go financed pension systems. Journal of Institutional and Theoretical Economics 145, 643-658.

Burggraf, Shirley P., 1993. How should the cost of child rearing be distributed? Challenge 36, 48-55.

Cigno, Alessandro, 1993. Intergenerational transfers without altruism. Family, market and state. European Journal of Political Economy 9, 505-518.

Cremer, Helmuth, Gavari, Firouz, Pestieau, Pierre, 2008. Pensions with heterogeneous individuals and endogenous fertility. Journal of Population Economics 21, 961-981.

Cremer, Helmuth, Lozachmeur, Jean-Marie, Pestieau, Pierre, 2004. Social security and variable retirement schemes. An optimal income taxation approach. Journal of Public Economics 88, 2259-2282.

Cremer, Helmuth, Lozachmeur, Jean-Marie, Pestieau, Pierre, 2010. Collective annuities and redistribution. Journal of Public Economic Theory 12, 23-41.

de la Croix, David, Doepke, Matthias, 2003. Inequality and growth: why differential fertility matters. American Economic Review 93, 1091-1113.

Eckstein, Zvi, Wolpin, Kenneth I., 1985. Endogenous fertility and optimal population size. Journal of Public Economics 27, 93-106.

Folbre, Nancy, 1994. Children as public goods. American Economic Review 84, Papers and Proceedings, 86-90.

Galor, Oded, Weil, David N., 1996. The gender gap, fertility, and growth. American Economic Review 86, 374-387.

Groezen, Bas van, Leers, Theo, Meijdam, Lex, 2003. Social security and endogenous fertility: pension and child allowances as Siamese twins. Journal of Public Economics 87, 233-251.

Samuelson, Paul A., 1975a. The optimal growth rate of population. International Economic Review 16, 531-538.

Samuelson, Paul A., 1975b. Optimum social security in a life-cycle growth model. International Economic Review 16, 539-544.

Sinn, Hans-Werner, 2004. The pay-as-you-go pension system as a fertility insurance and enforcement device. Journal of Public Economics 88, 1335-1357.

Yakita, Akira, 2001. Uncertain lifetime, fertility and social security. Journal of Population Economics 14, 635-640.

Zhang, Junsen, Zhang, Jie, Lee, Ronald, 2001. Mortality decline and long-run economic growth. Journal of Public Economics 80, 485-507.

Appendix 1: Existence and stability of the long-term equilibrium of α_t*

Substituting (9) into (6), we have

$$\alpha_t* = \frac{\rho}{\tilde{n}}[R(1-\tau)w\varepsilon\tilde{n} + (1-\sigma)\tau w \frac{\int_0^{\alpha_t*} dF(\alpha) + (1-\varepsilon\tilde{n})\int_{\alpha_t*}^{\overline{\alpha}} dF(\alpha)}{\int_0^{\alpha_{t+1}*} dF(\alpha) + \sigma\int_{\alpha_{t+1}*}^{\overline{\alpha}} dF(\alpha)} \tilde{n}\int_{\alpha_t*}^{\overline{\alpha}} dF(\alpha) \quad \text{(A1)}$$

according to which the cut-off degree α_t* changes from period to period. Using the relation $\int_a^b dF(\alpha) = F(b) - F(a)$, $F(0) = 0$ and $F(\overline{\alpha}) = 1$, the dynamic equation (A1) can be rewritten as

$$\alpha_t^* = \frac{\rho}{\tilde{n}}[R(1-\tau)w\varepsilon\tilde{n} + (1-\sigma)\tau w \frac{F(\alpha_t^*) + (1-\varepsilon\tilde{n})(1-F(\alpha_t^*))}{F(\alpha_{t+1}^*) + \sigma(1-F(\alpha_{t+1}^*))} \tilde{n}(1-F(\alpha_t^*))]. \quad \text{(A1')}$$

Defining $\Gamma(\alpha)$ as

$$\Gamma(\alpha) \equiv \alpha - \frac{\rho}{\tilde{n}}[R(1-\tau)w\varepsilon\tilde{n} + (1-\sigma)\tau w \frac{F(\alpha) + (1-\varepsilon\tilde{n})(1-F(\alpha))}{F(\alpha) + \sigma(1-F(\alpha))} \tilde{n}(1-F(\alpha))],$$

the steady state of α_t* is given by a solution of $\Gamma(\alpha) = 0$. We have

$$\Gamma(0) = -\frac{\rho}{\tilde{n}}[R(1-\tau)w\varepsilon\tilde{n} + (1-\sigma)\tau w \frac{(1-\varepsilon\tilde{n})}{\sigma}\tilde{n}] < 0, \quad \text{and}$$

$$\Gamma(\overline{\alpha}) = \overline{\alpha} - \frac{\rho}{\tilde{n}}[R(1-\tau)w\varepsilon\tilde{n}].$$

When condition (5) is satisfied, $\Gamma(\overline{\alpha}) > 0$. Assuming $F(\alpha)$ is continuous, $\Gamma(\alpha)$ is continuous. So, by the intermediate value theorem, there exists $\alpha*$ such that $\Gamma(\alpha*) = 0$, i.e., a steady state exists.

The above arguments do not rule out the possibility of multiplicity of the steady states. But we will assume the steady state is unique for our analytical convenience.

Next, we discuss the stability of the steady state. Differentiating (A1') at the steady state, we have

$$\frac{d\alpha_{t+1}*}{d\alpha_t*} = \frac{\tau w\rho(1-\sigma)f(\alpha*)[\frac{2\varepsilon\tilde{n}(1-F(\alpha*))-1}{F(\alpha*)+\sigma(1-F(\alpha*))}]-1}{\tau w\rho(1-\sigma)[\frac{[F(\alpha*)+(1-\varepsilon\tilde{n})(1-F(\alpha*))](1-F(\alpha*))(1-\sigma)f(\alpha*)}{[F(\alpha*)+\sigma(1-F(\alpha*))]^2}]}. \quad \text{(A2)}$$

While the denominator on the right-hand side of (A2) is clearly positive, the sign of the numerator cannot be determined a priori. But, we can see that, assuming $\varepsilon = 0.075$ as in de la Croix and Doepke (2003), $2\varepsilon\tilde{n}(1-F(\alpha*))-1$ is positive only when $\tilde{n} > 6$. It is plausible that $\tilde{n} \leq 6$, especially in developed countries. So, we assume that $2\varepsilon\tilde{n}(1-F(\alpha*))-1$ is negative, and hence, that the right-hand side of (A2) is negative. Then, the steady state is stable if

$$-1 < \frac{\tau w\rho(1-\sigma)f(\alpha*)[\frac{2\varepsilon\tilde{n}(1-F(\alpha*))-1}{F(\alpha*)+\sigma(1-F(\alpha*))}]-1}{\tau w\rho(1-\sigma)[\frac{[F(\alpha*)+(1-\varepsilon\tilde{n})(1-F(\alpha*))](1-F(\alpha*))(1-\sigma)f(\alpha*)}{[F(\alpha*)+\sigma(1-F(\alpha*))]^2}]} < 0 \quad \text{(A3)}$$

is satisfied. In the analysis of the paper, we assume the stability condition (A3) is satisfied.

Appendix 2: Illustration of equilibrium

We illustrated the image of the equilibrium changes in the cut-off degree of preference and the social security benefit with respect to an increase in the contribution. The equilibrium is given by equations (10) and (11):

$$\frac{d\alpha^*}{d\tau} = \frac{\rho}{\tilde{n}}[-Rw\varepsilon\tilde{n} + (1-\sigma)\frac{d\beta_{t+1}}{d\tau}] \tag{10}$$

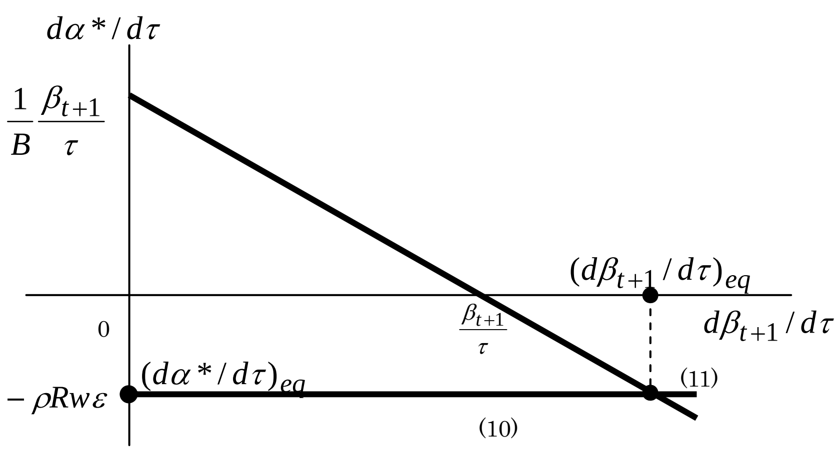
$$\frac{d\beta_{t+1}}{d\tau} = \frac{\beta_{t+1}}{\tau} - B\frac{d\alpha^*}{d\tau} \tag{11}$$

where

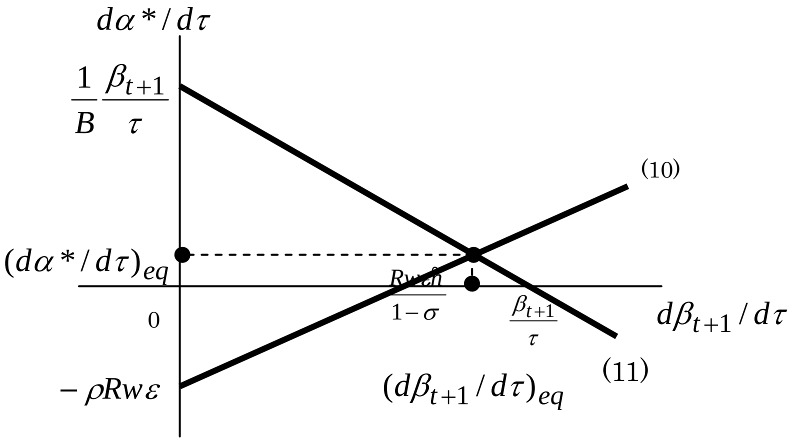
$$B \equiv \beta_{t+1}\frac{(1-\sigma)f(\alpha^*)}{\int_0^{\alpha^*}dF(\alpha) + \sigma\int_{\alpha^*}^{\overline{\alpha}}dF(\alpha)} + \frac{\tau w\tilde{n}f(\alpha^*)}{\int_0^{\alpha^*}dF(\alpha) + \sigma\int_{\alpha^*}^{\overline{\alpha}}dF(\alpha)}\{[\int_0^{\alpha^*}dF(\alpha) + (1-\varepsilon\tilde{n})\int_{\alpha^*}^{\overline{\alpha}}dF(\alpha)] - \varepsilon v_t\} > 0 . \tag{A1}$$

We have four cases: (i) $\sigma = 1$; (iia) $\sigma < 1$ & $(1-\sigma)\frac{\beta}{R} > \tau w\varepsilon\tilde{n}$; (iib) $\sigma < 1$ & $(1-\sigma)\frac{\beta}{R} = \tau w\varepsilon\tilde{n}$; (iic) $\sigma < 1$ & $(1-\sigma)\frac{\beta}{R} < \tau w\varepsilon\tilde{n}$. These cases can be illustrated as follows:

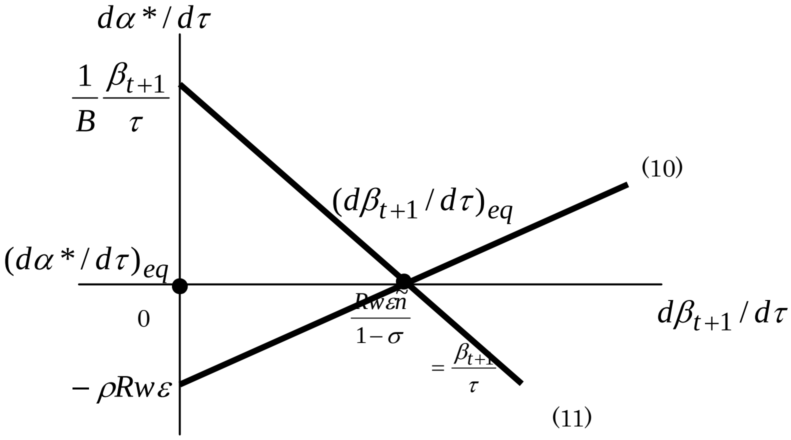
Case (i) $\sigma = 1$



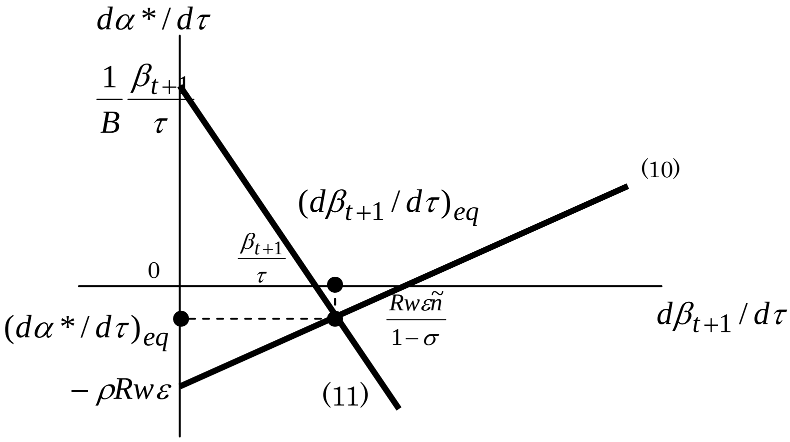
Case (iia) $\sigma < 1$ & $(1-\sigma)\beta / R > \tau w \varepsilon \tilde{n}$



Case (iib) $\sigma < 1$ & $(1-\sigma)\beta / R = \tau w \varepsilon \tilde{n}$



Case (iic) $\sigma < 1$ & $(1-\sigma)\beta / R < \tau w \varepsilon \tilde{n}$



where $(d\alpha^* / d\tau)_{eq}$ and $(d\beta_{t+1} / d\tau)_{eq}$ denote the values at the equilibrium satisfying (10) and (11). From these figures, we can show that when $\sigma = 1 - \varepsilon\tilde{n} < 1$ and $d\alpha^*/d\tau > 0$ as in case (iia), we have $\frac{\beta_{t+1}}{\tau} > \frac{d\beta_{t+1}}{d\tau} > \frac{Rw\varepsilon\tilde{n}}{1-\sigma} = Rw$, i.e., $-Rw + \frac{d\beta_{t+1}}{d\tau} > 0$, and that when $\sigma = 1 - \varepsilon\tilde{n} < 1$ and $d\alpha^*/d\tau < 0$ as in case (iic), we have $\frac{\beta_{t+1}}{\tau} < \frac{d\beta_{t+1}}{d\tau} < \frac{Rw\varepsilon\tilde{n}}{1-\sigma} = Rw$, i.e., $-Rw + \frac{d\beta_{t+1}}{d\tau} < 0$.

Equation (11) is the combinations of $(d\beta_{t+1}/d\tau, d\alpha^*/d\tau)$ satisfying the budget equation of the social security authority. When $\sigma = 1 - \varepsilon\tilde{n} < 1$ and $d\alpha^*/d\tau > 0$ as in case (iia), a decrease in the contribution rate increases the number of individuals who have children. Since the labor supply and the aggregate contribution decline largely, the negative effect on benefit will be great. In contrast, when $\sigma = 1 - \varepsilon\tilde{n} < 1$ and $d\alpha^*/d\tau < 0$ as in case (iic), a decrease in the contribution rate decreases the number of individuals who have children. In this case, the labor supply and the aggregate contribution increases, so the negative effect on the benefit will be smaller. Thus, in case (iic), since an increase in the contribution rate reduces the labor supply and the aggregate contribution to the social security system, it does not increase the benefit greatly and hence lowers the utility levels of individuals. On the other hand, in case (iia), increases in the contribution rate increase the labor supply and, therefore, greatly increase the benefit. The increased benefit increases the utility levels of individuals.